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THE SABINE EQUATION AND COUPLED SPACES IN CHURCHES

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INTRODUCTION

The reverberation time equations have been the most widely used prediction tools in acoustical design because they are simple to use and usually give reasonable results. The first and perhaps the most widely used reverberation time equation is the Sabine equation. The purpose of this study was to test the use of the Sabine equation in churches especially when recesses and coupled spaces are present.

The main investigation is focused on the Roman Catholic churches of Portugal. Portugal is one of the oldest European countries and played a prominent role in some of the most significant events in world history. It presents an almost perfect location to trace the history of Catholic church buildings in the world. Portuguese churches can be considered a representative example of Catholic churches in the world.

This study reports on acoustical field measurements in a major survey of 41 Roman Catholic churches in Portugal that were built between the 6th century and 1993 (Carvalho 1994). The churches were selected to represent the main architectural styles found throughout Portugal and to represent the evolution of its 14 centuries of church construction. Acoustical measurements were taken in: 12 *Visigothic* or *Romanesque* churches (6th-13th centuries), 16 *Gothic* or *Manueline* churches (13th-16th centuries), 13 *Renaissance, Baroque* or *Neoclassic* churches (16th-19th centuries) and 4 *Contemporary* churches (20th century). The main architectural features of the churches are displayed in Table 1.

The method used to calculate the Reverberation Time (RT30) is based on the integrated impulse-response method. A limited-bandwidth noise-burst is generated and transmitted into the church by a loudspeaker via an amplifier. The room's response to the noise-burst is then sampled from the RMS detector output of the sound level meter. The receiving section consisted of one 1/2" microphone (five receiver positions were, on average, used, depending on the width of the church) and a sound level meter with a filter set. The loudspeaker was placed at two sound source locations in each church: one in front of the altar and another in the center of the main floor (at 0.8 m above the floor and at a 45° angle with the horizontal plane). The procedure was commanded by a specific control software. Each measurement was calculated from an ensemble of three and four pulse responses in each position. The microphone at each location was placed at 1.30 m above the floor. In total, nearly 8000 values were determined (all combinations of frequency bands and source/receiver locations). The churches were measured while unoccupied.

ANALYSIS BETWEEN *RT REAL* AND *RT EXPECTED* (SABINE EQUATION)

In this study the Sabine equation for the prediction of RT was applied to the 41 churches measured:
$RT = 0.16\ V / A$, where V-Volume (m^3), RT-Expected Reverberation Time (s) and A-Total Absorption (m^2).

The measured RT values (*RT real*) were compared with the predicted values using the Sabine equation using only freq. = 500 and 1000 Hz in the averaging method. The Sabine equation did not provide a very good prediction of the measured RTs. Linear regression models between RT_{REAL} and RT_{SABINE} displayed Pearson correlation coefficients of approximately 0.73. The results for the application of the Sabine equation to this sample of churches showed also huge differences between measured and estimated RTs (Avg $|RT_{REAL} - RT_{SABINE}|_i$ = 71%). The differences are due to the presence, in some churches, of chapels and other deep spaces that act as coupled spaces. Therefore the importance of the coupled spaces justified the search for a new approach in using the Sabine equation in these situations.

COUPLED SPACES

The subdivision of the volume into a number of smaller volumes coupled together, results in very low RT without the addition of absorptive materials. Deep lateral chapels and even in certain cases, the main altar area (apse), can act as coupled spaces. This will entirely transform the analysis and application of the prediction equation. The border between those coupled spaces and the main room acts as an absorptive surface with an indeterminate absorption coefficient α.

Cremer states, as a *rule of thumb*, that if the boundary area covered with absorptive materials in the coupled room (Sa) exceeds that of the coupling area to the main room (Sc), it should be treated as an open window (α=1); if not, the coupled room (room 2) should be treated as part of the main room. Using that rule and considering that all chapels and the main altar area have at least a Sa=Sc due to the wood-carved altars that fill one of the walls entirely and freely supposing that the wood-carving is an absorptive material a spreadsheet was calculated using an α=0.9 in all openings to chapels or to the main altar area. This approach did not produce satisfactory results (Avg $|RT_{REAL} - RT_{SABINE}|_i$ = 35%).

The answer seems to indicate the use of different α's for the main altar area and for the lateral chapels. In many of the churches, the chapels can not be considered as coupled rooms due to their size or shape. As Kuttruff states, the necessity of considering coupling effects when calculating the RT arises if the area of the coupling aperture is substantially smaller than the total wall area of a partial room. Another explanation can be in the lack of diffusion that happens in some of the churches, especially those having very simple geometric shapes and extremely non-uniform distribution of absorption on their walls.

SABINE EQUATION - NEW ALGORITHM WITH COUPLED SPACES

Method. Lateral chapels, the main altar (apse) and lateral aisles, can in certain cases act as coupled spaces (CS). This will entirely transform the analysis and application of the Sabine equation. A new algorithm for use in the Sabine equation considering the existence of CS was developed. An absorption coefficient for the opening of each coupled space (α_{CS}) was calculated depending on the geometric characteristics of the specific CS. With that α_{CS} a new Total Absorption for the church was calculated and the Sabine equation was used with the appropriate Final Volume (*Volume Total* was used if no coupled spaces and *Volume Nave* was used if chapels and main altar are coupled spaces, etc.). RT_{SABINE} = 0.16 V. Final / A, where: V - Volume (m^3), α_{CS} - Absorption coefficient (coupled space), A - Total absorption (m^2) = $\Sigma A_i + \Sigma \alpha_{CS\,j} S_j$, S - Coupled space opening surface area (m^2).

The necessity of considering coupling effects when calculating the RT arises if the area of the coupling aperture is substantially smaller than the total wall area of the partial (or coupled) room. Therefore, a geometrical parameter was found to *weight* the *degree of coupling* of a specific partial room to the main room volume. Using Fig. 1 (where *l*, *w* and *h* are the length, width and height), *l/w* appears as a good parameter to characterize a coupled space. Then, $\alpha_{CS} = f(l/w)$. This function *f* must be restricted to the limits of α_{CS}. That is, it must be between 0 and 1. The TANH (hyperbolic tangent) was chosen with an *x* axis shift to eliminate the presence of negative α_{CS}'s. Therefore, the final transfer function is: $\alpha_{CS} = \tanh [\, a\,(l/w - b)\,]$

Table 2 presents the best parameters *a* and *b* that were found by experimentation, using the 41 church sample. Other general rules in the use of this algorithm are presented below:

CHAPELS are only considered as coupled spaces if $l/w > 0.6$. l/w is the average of all $(l/w)_{chapel\ i}$ weighted by their opening surfaces S_i. This is the area of the vertical plan that is the border between the chapel and the main volume of the church. The total interior absorption should be included. In the simplified version of this method, this absorption is sufficient in the account of the total absorption for this type of coupled space. If the chapels are *inside* the lateral aisles area, they should be omitted if that volume is also omitted as referred below if $l/w_{\,(lateral\ aisles)} > 0.70$.

MAIN ALTAR (APSE) is only considered as coupled spaces if $l/w > 0.6$. The total interior absorption should be accounted for (normally this is a very small quantity).

LATERAL AISLES are only considered as coupled spaces if $l/w > 0.4$. In this type of coupled space the parameters *l* and *w* are defined as seen in Fig. 2 where *l* = width of lateral aisle and *w* = height of each opening. The volume of the Lateral Aisles is only excluded of the Total Volume of the church if $l/w > 0.70$:
Volume Final = Volume Nave - Volume Lateral Aisles if $l/w > 0.70$ or
Volume Final = Volume Nave if $l/w \leq 0.70$. The total interior absorption should be included.

Results. The results of this algorithm applied to the 41 churches are displayed in Fig. 3. An average of 16% between measured and predicted RT was found for the total 41 churches. This is a huge improvement from the 71% average absolute difference found without the use of this algorithm. Using *seconds*, the average of the absolute differences is 0.49 s in the RT expected, which can be considered a very good result due to the large values for the RTs involved.

Fig. 3 presents the plot of the RT_{REAL} vs $RT_{SABINE\ (w/\ CS)}$ and the prediction line. This prediction linear equation ($RT_{REAL} = -0.003 + 0.999\ RT_{SABINE}$) with R = 0.89 is very close to the ideal ($RT_{REAL} = RT_{SABINE}$).

The *Maximum Height* appeared as a justification for part of the differences found between RT_{REAL} and RT_{SABINE} in a general linear model to predict the RT_{REAL} with the use of the RT_{SABINE} together with 15 architectural parameters. With an α-to-enter/remove = 0.15 the result of a linear model was:

$$RT_{REAL} = -0.162 + 0.835\ RT_{SABINE} + 0.048\ MAX\_HEIGHT \qquad (R^2 = 0.81)$$

This supports the explanation that the RT differences are due to the lack of diffusion that occurs in some of the churches, especially those having simple geometric shapes and extremely non-uniform distribution of absorption on their walls. This occurs in rectangular churches with smooth, reflecting walls and a tall ceiling. The absorption is mainly concentrated on the ceiling if it is wood or/and on the floor if it is wood or if wooden pews are used. In this case a two-dimensional reverberant sound field can be built.

Generally, the higher the ceiling, the longer the RT. The higher ceiling can almost act as a *reverberant chamber* included in the main room. This will only happen if the ceiling is non absorptive, that is, if it is not made of wood (in this sample of churches). To check this hypothesis the 41 churches were grouped according to their ceiling type (*wood* and *non wood*). The Pearson correlation coefficients were then calculated between these two groups and the ΔRT. The results are found in Table 3.

Figure 4 shows the RT differences grouped by the two groups of ceiling type with the standard error interval. An ANOVA test was calculated to determine if these two groups of ceiling types were statistically different. It was found that, at a level of probability (p-value) higher than 0.12, the two groups were statistically different. Therefore it can be concluded that there are enough data to support the idea that a *reverberant ceiling effect* may play a role in the differences found between the RT real and the RT calculated by the Sabine equation.

Simplified Method. A simplified method of the new algorithm presented above is now described. The α_{CS} of the CHAPELS should be equal to 0. The interior absorption of each chapel is normally sufficient to consider the effect of chapels in the overall absorption of the church. Therefore an $\alpha_{CS\ (CHAPELS)} = 0$ can be used as a simplification. The α_{CS} of the LATERAL AISLES should be equal to 0.17. The Lateral Aisles (LA) have very similar proportions relatively to the church main volume. Therefore an $\alpha_{CS\ (LATERAL\ AISLES)} = 0.17$ can be used as a simplification if $l/w > 0.4$. Then the Main Altar area (MA) will be the only coupled space to be considered if $l/w > 0.60$ in this simplified version of the algorithm presented: $A_{CS} = 0 \,.\, S_{CH} + 0.17\ S_{LA} + \tanh\,[0.985\,(\,l/w - 0.6\,)]$.

SUMMARY

The use of the Sabine reverberation time equation was tested to estimate the measured reverberation times in a 41 church sample. The Sabine equation was not efficient in predicting the RT when the effect of coupled spaces (CS) is not considered. The effect of CS was analyzed and a new algorithm for the application of the Sabine equation in churches was developed producing an average of 16% in the differences between the real and predicted RTs compared to a 71% difference using the standard Sabine equation. CS were found to act as *windows* with a characteristic α depending on their dimensions. The recesses in churches were grouped in three types: main altar area (apse), chapels and lateral aisles. Each type of CS has a particular acoustical behavior with different *a* and *b* parameters in the calculated equation. There are two major reasons that three types of CS are needed. The first reason is the relative position of the sound source to the CS, that is, concerning the direction from which the sound enters the CS. Second is the volume of the CS relative to the volume of the main room. It was found that those recesses only acted as CS if their *length / opening_width* > 0.6 or if the *aisle_width / opening_height* > 0.4 in lateral aisles. The remaining differences found between the measured RTs and the predicted RTs with this new algorithm were hypothesized to be related to what was called a *reverberant ceiling effect* which is presumed to be due to a two-dimensional reverberant sound field that builds up near a very tall ceiling in churches.

REFERENCES

- Carvalho, António P.O. *Influence of Architectural Features and Styles on Various Acoustical Measures in Churches,* PhD. Diss. U. Florida 1994
- Cremer, L., and H. Müller. *Principles and Applications of Room Acoustics*, English transl. T. J. Schultz. Vol. 1 (Applied Science, England, 1978)
- Kuttruff, Heinrich. *Room Acoustics* (Applied Science, London, 1991)

TABLE 1 - Simple statistics for all churches tested.

ARCHIT. FEATURE	Min.	Max.	Mean	Median
VOLUME (m^3)	299	18674	5772	3918
AREA (m^2)	56	1031	450	427
MAX. HEIGHT (m)	7	39	15	13
MAX. LENGTH (m)	12	62	33	31
WIDTH NAVE (m)	4	38	13	11

TABLE 2 - Coeff. (*a* and *b*) to use in new algorithm to account for the coupled spaces effect in the use of the RT Sabine equation.

TYPE OF COUPLED SPACE	*a*	*b*
CH - CHAPELS	0.007	0
MA - MAIN ALTAR (APSE)	0.985	0.6
LA - LATERAL AISLES	0.0118	-14

TABLE 3 - Pearson correlation coefficients between ΔRT and *Height maximum*.

TYPE OF CEILING	NUMBER OF CHURCHES	R	
		ΔRT in sec.	ΔRT in %
WOOD	22	0.030	-0.004
NON WOOD	19	-0.216	-0.154

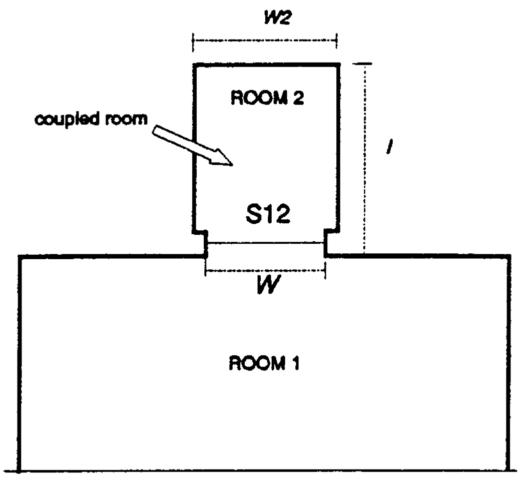

Fig. 1 - Plan sketch of a general church with a coupled space. *l*-length, S_{12}-opening surface area, *w*-opening width, *w*2-coupled space width, Room 1-main room, Room 2-coupled room.

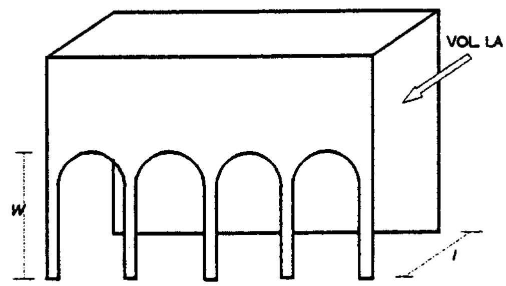

Fig. 2 - 3-D sketch of lateral aisles in a general church. *l*-width of lateral aisle, *w*-width of opening to lateral aisle, vol_{LA}-volume of lateral aisle.

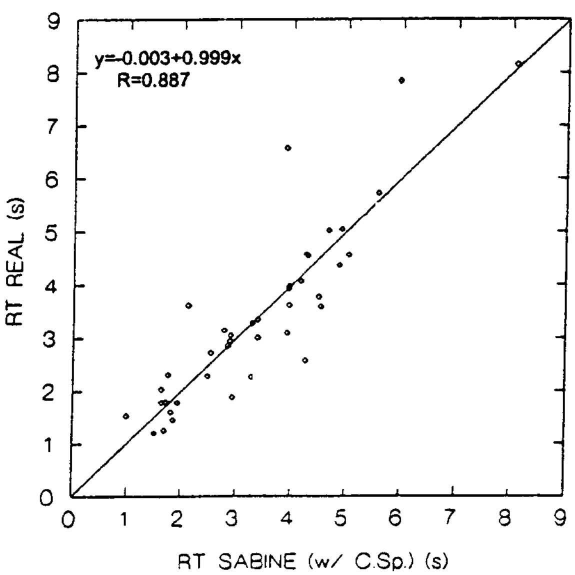

Fig. 3 - Plot of measured (*y axis*) vs. predicted (*x axis*) RTs with linear prediction line and Pearson correlation coefficient

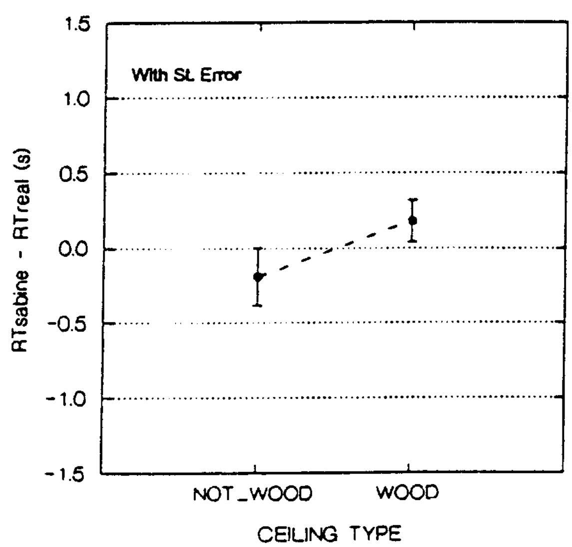

Fig. 4 - Analysis of the effect of ceiling type (*wood* and *non wood*) differences (RT_{REAL} - RT_{SABINE}). Mean values for all the churches in each ceiling type are shown with one stand. error confidence interval.