A Truncated Fourier Series with Genetic Algorithm for the control of Biped Locomotion

Nima Shafii, Mohammad H. Seyed Javadi, Bahram Kimiaghalam

Abstract— **Humanoid research has made notable progress during the past 25 years. However, currently most humanoids use the ZMP (Zero Moment Point) for control of bipedal locomotion, which requires precise modeling and actuation with high control gains. On the contrary, researchers do not rely on such precise modeling and actuation. In this paper we have tried to introduce a novel method for the evolution of walking behavior in a simulated humanoid robot with up to 22 degrees of freedom. In this method a modified Truncated Fourier Series (TFS) generates walking angular trajectories and it is optimized by genetic algorithm. By studying human walking TFS parameters have also been reduced. As a test-bed, we chose Robocup 3D soccer simulation environment (spark) and implemented our method in MRL 3D team's agents. Experimental results show that training of the robot can be successfully performed by our method, thus allowing the biped robot to walk fast, stably and straightly.**

I. INTRODUCTION

In recent years, bipedal locomotion, especially "bipedal walking" has been one of the interesting research topics in multi disciplinary topic. Bipedal walking as a very complex motion, involves most of humanoid joints including its sensors and actuators. Many researchers have focused on this topic and a lot of approaches have been presented. But so far no method exists that can walk a robot as stable as human's do. There are two major approaches in bipedal walking; model-based and model free approaches. In model-based approach the designer first derives model of the robot and then builds a controller for the model. Two well known methods in this approach are "Zero Moment Point" [1] (ZMP) and "Inverted Pendulum" [2].

In model-free approach, which is also called "Dynamics Based", it is common to make use of the sensory information and associate it with motions. No physical model is used in this method that eases the implementation of the skills. There are three important studies done in this field; Passive Dynamic Walking (PDW) [3], Central Pattern Generator (CPG) [4] and Ballistic Walking [5]. In PDW approach, the robot does not have any actuators model and it walks just by utilizing the gravity force. The Ballistic walking is originated from the simple human walking model based on the observation of human walking in which the muscles of the swing leg are activated only at the beginning and the end of the swing phase. In CPG approach, special neural circuits take the role of rhythmic walking controller using the non-linear equations to model the neural activities. Researchers usually focus on complex mathematical models like Hopf [6] or Matsuoka [7] to model these neural activities and generate rhythmic walk patterns (Gait).

In 2006, Truncated Fourier Series (TFS) formulation is used for gait generation in bipedal locomotion [8]. TFS together with a ZMP stability indicator are used to prove that TFS can generate suitable angular trajectories for controlling bipedal locomotion. It does not require inverse kinematics and Stable gaits with different step lengths and stride frequencies can be readily generated by changing the value of only one parameter in the TFS.

We have designed a novel method and Gait generator that is fast and efficient to implement different humanoid locomotion skills. For the first time in this paper, modified TFS is implemented in a simulated humanoid robot and TFS parameters are also reduced by 2 dimensions (down to 6 dimensions) as a gait generator then optimized by Genetics Algorithm (GA) to achieve a fast and robust bipedal walking.

II. BIPED MODEL AND SIMULATOR

In this paper, a new approach for walking behavior in a simulated humanoid robot is discussed. However simulation is not always efficient, due to difficulty of modeling the collision between feet and the ground, yet we believe that numerical simulation is sufficient to explore and test bipedal locomotion methods.

The simulation is performed by Rcssserver3d [9] simulator which is a generic three-dimensional simulator based on Spark and Open Dynamics Engine (ODE). Spark is capable of carrying out scientific distributed multi agent calculations as well as various physical simulations ranging from articulated bodies to complex robot environments. The robot in this study is a simulated model of NAO Robot that is a real humanoid Robot with two arms, two legs and a head. This robot weighs 4.5kg, stands 57cm high and has 22 degrees of freedom (DOF). There are six DOFs in each leg; two in the hip, two in the ankle and one at the knee. An additional DOF that exists at each leg's hip for yaw causes the legs to rotate outward and inward.

It is found that 6 DOFs (three for each leg) are more important than other DOFs in fast walking. These are DOFs of hip; knee and ankle DOFs which move on the same plane of forward-backward. Although other DOFs are effective in walking behavior, but in fact, their role is more in smoothing

---

S. N. is with the Young Researchers Club at Islamic Azad University, Qazvin Branch, Iran. (e-mail: shafii@mrl.ir).

H.S.J M. is with the Young Researchers Club at Islamic Azad University, Qazvin Branch, Iran. (e-mail: seyedjavadi@mrl.ir).

K. B. is vice president of research, Islamic Azad University, Qazvin Branch, Iran. (e-mail: bahkim@yahoo.com).

the robots walking motion. So here, it's preferred to ignore them to decrease learning search space. In this work, similar to [10], Foot was kept parallel to the ground by using ankle joint in order to avoid collision. Therefore ankle trajectory can be calculated by hip and knee trajectories and ankle DOF parameters are eliminated.

III. TFS GAIT GENERATOR

Bipedal walking as a complex motion, involves most of humanoid robot's joints. Researchers attempt to imitate the human walking style as well as its speed. Therefore analyzing human walk pattern has been used for acquiring beneficial information about this motion. Human walk has been investigated from many aspects; walking trajectory being one of them. The walking trajectory is divided into several types. Positional trajectory and angular trajectory are two of them. In angular trajectory, the angle of each joint is plotted at a certain time slice. Therefore the angular trajectory is obtained by angular variation of each joint. Angular trajectory of two joints; hip and knee captured from human walking are shown in Fig. 1.a. [11]

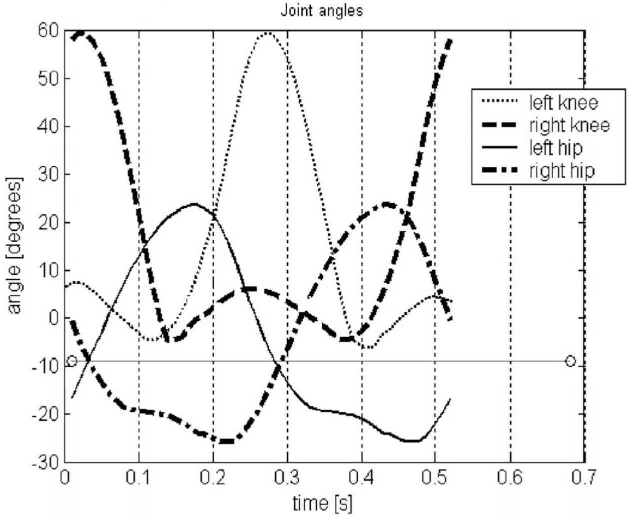

Fig. 1.a Human walking angular trajectory

The angle of each joint in one period of walking signal from t_0 to t_6 is represented in fig. 1.b [11]. Capturing the main aspects of fig. 1.a gives us a general form that is applicable to robots. In time frames $[t_0,t_2]$ and $[t_5,t_6]$ the left leg is support leg and the right leg is swing leg, but in time frame $[t_2,t_5]$ the left and right legs play the role of support and swing legs respectively. In other words, at times of t_2 and t_5 the roles of the legs are switched. At time t_3 where two hip trajectories intersect, two thighs cross each other.

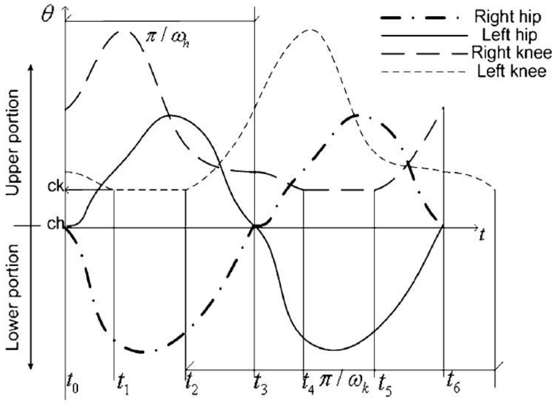

Fig. 1.b gaits elaborated from human gaits features

A. Angular trajectory generation:

Due to the fact that all joint's trajectories of human walking are periodic and similar to sine or cosine signals [12], the generation of these angular signals can be done by Fourier series.

B. Basic Fourier series

The original definition of Fourier series is described by following formula:

$$F(t)=\frac{a_0}{2}+\sum_{i=1}^{\infty}\left(a_i\cos\frac{i\pi t}{L}+b_i\sin\frac{i\pi t}{L}\right) \quad (1)$$

The first term ($\frac{a_0}{2}$) of (1), represents the DC bias of the signal and the L represents half of the largest period that exists in the signal. $w=\frac{\pi}{L}=\frac{2\pi}{T}$ then the frequency form of Fourier series is as follows:

$$F(t)=\frac{a_0}{2}+\sum_{i=1}^{\infty}\left(a_i\cos(iwt)+b_i\sin(iwt)\right) \quad (2)$$

Where w is fundamental frequency of periodic signal content. Any complicated signal can be produced by this formula when i is considered infinite. When the value of i is limited to a definite number, precision of generating signal is reduced and this type of Fourier series is called partial sum of the Fourier series. According to fig. 1.a, human walking angular trajectories are too complicated to be produced by a definite Fourier series band that is limited only to the second harmonic. Therefore a modified definite Fourier series namely a Tru ncated Fourier series (TFS) is used in this study.

A. Trajectory generation by using TFS

According to Fig. 1.b, the signals are divided in two parts; the upper portion and the lower portion. Since each portion can be assumed as an odd function, the cosine part of Fourier series is eliminated and reduced to the following for generating each portion of the trajectory.

$$F(t) = a + \sum_{i=1}^{n} b_i \sin(iwt) \quad (3)$$

Where w is fundamental frequency of the signal and a is the signal offset. Separate Fourier series for each portion, allows generating more complex signals with different upper and lower parts. Also the number of parameters for generating this complex signal is less comparing to when the partial sum of Fourier series is used to produce the whole signal.

As shown in Fig. 1.b, each signal has an offset. C_h is offset of hip trajectory and C_k is offset of knee trajectory. From t_1 to t_2 the left leg is considered as supporting leg and the variation of its knee angle is so minute that can be assumed fixed. This duration of walking is named knee lock phase. In addition, the amount of shift phase of the two leg trajectories signal is as half of the period of each signal so by producing trajectory of one leg the other leg's trajectory can be calculated. The trajectories for both legs are identical in shape but are shifted in time relative to each other by half of the walking period. By using (3), and considering curves of Fig. 1.b, the TFS for generating each portion of hip and knee trajectories are formulated as below:

$$\begin{aligned}
&\theta_h^+ = \sum_{i=1}^{n} A_i.\sin(iw_h t) + c_h, w_h = \frac{2\pi}{T_h} \\
&\theta_h^- = \sum_{i=1}^{n} B_i.\sin(iw_h t) + c_h, w_h = \frac{2\pi}{T_h} \quad (4) \\
&\theta_k^+ = \sum_{i=1}^{n} C_i.\sin(iw_k t) + c_k, w_k = w_h \\
&\theta_k^- = c_k \geq 0
\end{aligned}$$

In these equations, the plus (+) sign represents the upper portion of walking trajectory and the minus (-) shows the lower portion. $i = 1$ and A_i, B_i, C_i are constant coefficients for generating signals. The h and k index stands for hip and knee respectively. Also c_h, c_k are signal offsets and T_h is assumed a period of hip trajectory. Considering the fact that all joints in walking motion have equal movement frequency and stride rates is statistically equal [12], the equation $w_k = w_h = \frac{2\pi}{T_h}$ can be concluded. Parameter t_1 is the start time of lock phase for knee joint and parameter t_2 represents the end time of lock phase and in this duration of time $\theta_k^- = c_k \geq 0$.Therefore truncated Fourier series parameters are $c_h, c_k, A_i, B_i, C_i, t_1, t_2, w_h$ And an optimization algorithms must optimize the 8 Dimensions Problem to find the best gait generator in this stage.

B. Proposed model for reducing parameters

In robotics studies, researchers have always attempted to imitate human walk for legged locomotion, so analysis of human walk is inevitable. The relationship between human joints and its angle trajectories during a walking cycle is a suitable approach for this goal. Fig. 2 shows the angular relationship between knee and hip through human walk [13].

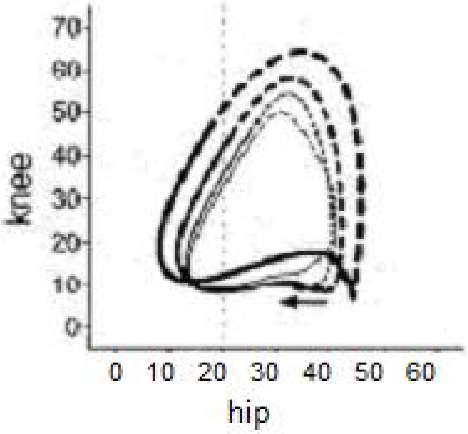

Fig. 2 Hip- Knee diagram in human walk

Since this paper has tried to use critical features involving human walking in order to model walking of a robot, any obtainable information about human walking is vital. We can get some useful information about knee lock phase by looking at fig. 2. According to this figure, straight line represents the knee lock phase and this phase starts when the hip angle reaches its maximum value. With the same logic, the lock phase is ended with minimum value of hip angle.

So by specifying the start and end time of lock phase, two parameters of t_1, t_2 could be eliminated. Therefore the number of variable for optimization decreased to 6. This omission has many advantages such as; reducing the search space of optimization problem and increasing the convergence speed of GA.

IV. GENETIC ALGORITHM FOR OPTIMIZING TFS

Bipedal walking is known as a complex motion because many factors affect Walking style and stability such as robot's Kinematics, collision between feet and the ground and dynamics of the robot. Therefore for this complex motion, relation between Gait trajectory and walking characteristic is nonlinear. Since this kind of optimization is usually difficult, a genetic algorithm is suitable to solve it.

GA is a stochastic searching procedure based on the mechanics of natural selection and genetics [14]. GA is used to find the best parameters to generate angular trajectories for bipedal locomotion.

Using GA in our optimization problem, parameters are coded in to a finite length of string (Genes) as a chromosome. According to section 2, TFS has 6 parameters to generate all joints angular trajectories, thus each chromosome has 6 Genes. In Designing of Chromosomes gene's type is considered as double format. Population for each generation is assumed to be 100. Chromosomes are generated randomly and uniformly for the first iteration between lower and upper bound. In this study the lower and upper bound data for initialization are depicted in following table.

TABLE I
LOWER BOUND & UPPER BOUND

	C_h	C_k	A	B	C	w_k
Upper Bound	30	0	45	0	0	1
Lower Bound	-10	-50	0	-45	-40	0.05

To make the robot learn how to walk, legs angular trajectories based on each chromosome are produced by TFS. Then each of these angular trajectories is used by simulated robot in walking. To follow angular trajectories, Proportional derivative (PD) controllers are used in all individual joints to attempt to drive towards their target angles. Finally, based on this walking, chromosome's fitness is calculated.

Fitness function has a critical role in GA and is used to judge how well a solution represented by a chromosome is. To achieve more stable and faster walk, a fitness function based on robot's straight movement with having limited time for walking is assumed. The amount of deviation from straight walking is subtracted from the fitness as a punishment to force the robot to walk straight. First the robot is initialized in x=y=0 to walk for 77 seconds then fitness function is calculated whenever robot falls or time duration for walking is over. Fitness function formulation is expressed as follow; The *Current Time* in the formula determines time passed since robot has started walking:

if ((*Current Time* >= *time duration for walking*) or (robot is fallen))

$$\text{Fitness: } = x - 0.5y \tag{5}$$

Fi

For using GA as an optimizer, scattered function has been chosen as a cross over function. This function creates a random binary vector with the length of chromosome. Then the genes where the produced vector is a 1 from the first parent and the genes where the vector is a 0 from the second parent has been selected, then by combining these genes the child will be formed.

For mutation, uniform function has been used where after selecting a fraction of chromosome with same probability as mutation rate (which is assumed as 0.06), a random number from range of upper and lower bound (table I) has been selected uniformly then existing number in the fraction has been replaced by this random number.

Selection method is roulette wheel and reproduction rate is assumed as 0.8. Termination condition is having a generation counter greater than 28. Therefore the GA requires 2800 trials to find appropriate TFS parameters.

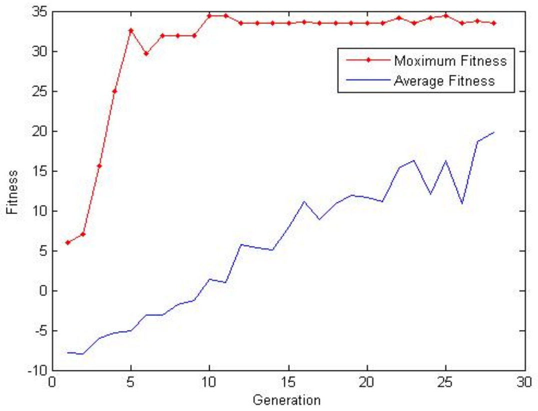

Fig. 3 Average fitness and Maximum fitness during 28 generations

After 9 hours from starting GA on a Pentium IV 3 GHz machine with 2 GB of physical memory, generation exceeded to 28 and the robot could walk straightly 34.5 m in 77 s with average body speed of around 0.45 m/s. The period time of each step was about 0.41 second. Fig. 3 shows the average and maximum fitness values for the robot over 28 generations. Although Elitism was employed, due to the fact that this simulation has inherent noise, Maximum fitness was falling as well as rising from generation to generation. In Fig. 4 angular trajectory generated by TFS after learning process and followed trajectory are shown.

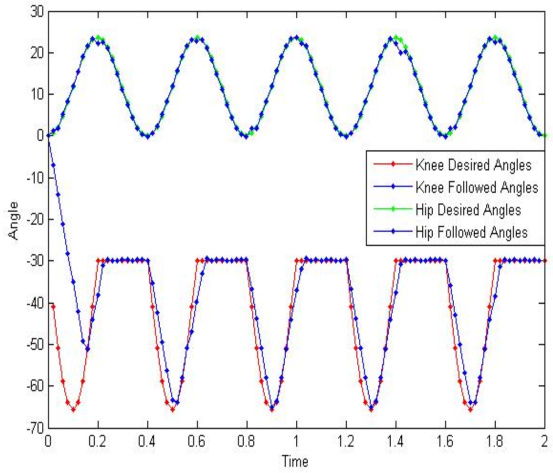

Fig. 4 Angular Trajectory generated by learned TFS (Desired angle) and followed one by controller (Followed angle) for left hip and left knee of simulated Nao robot.

Genetic algorithm led the robot to learn how to walk straight. Fig .5 shows the biped locomotion which is obtained from GA search.

Fig. 5. Learned Nominal trajectory is obtained by GA

V. *CONCLUSION*

This paper presents a model with 6 parameters for producing all walking trajectories. Since in this approach it is necessary to test sets of parameters on real or simulated robot, fewer dimensions is much better. Therefore the main advantage of this model is having the least parameters compare with other gait generators models. it is also capable of being implemented on any kinds of humanoid robots without considering its physical model. Considering the fact that this experiment needs fewer parameters, walking stability and speed can be improved by adding more gait generators to other joints such as hands.

REFERENCES

[1] M. Vukobratovic, B. Borovac, D. Surdilovic. “Zero-moment point proper interpretation and new applications”. *In Proceedings of the 2nd IEEE-RAS International Conference on Humanoid Robots, Humanoids 2001*, pp. 237–244 (Waseda University, Tokyo, Japan, 2001).

[2] S. Kajita, F. Kanehiro, K. Kaneko, K. Yokoi, H. Hirukawa “The 3D linear inverted pendulum mode A simple modeling for a biped walking pattern generation“ In: *Proceedings of the 2001 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pp. 239–246 (2001)

[3] S. H. Collins, M. Wisse, A. Ruina, "A 3-D passive dynamic walking robot with two legs and knees" *Int. J. Robot. Res.*, Vol. 20, pp. 607-615, 2001.

[4] C. Pinto, M. Golubitsky "Central Pattern Generator for Bipedal locomption" *J. Math. Biol.* ,Vol. 53, pp. 474–489, 2006, DOI 10.1007/s00285-006-0021-2.

[5] S. Mochon and T.A. McMahon “Ballistic walking”,*J. Biomech.*, Vol. 13, pp. 49-57,1980.

[6] J. Buchli, F. Iida, A.J. Ijspeert,"Finding Resonance: Adaptive Frequency scillators for Dynamic Legged Locomotion", *Proceedings of the 2006 IEEE/RSJ International Conference on Intelligent Robots and Systems,* pp. 3903-3910, 2006.

[7] K. Matsuoka. “Sustained oscillations generated by mutually inhibiting neurons with adaptation.” *Biol. Cybern.*. Vol. 52, pp. 367-376, 1985.

[8] L. Yang, C.M. Chew, A.N. Poo, “Adjustable Bipedal Gait Generation using Genetic Algourithm Optimized fourier Series Furmulation”, In : *Proceedings of the 2006 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pp. 4435-4440, Oct. 2006.

[9] J. Boedecker, “Humanoid Robot Simulation and Walking Behaviour Development in the Spark Simulator Framework,” *Artificial Intelligence Research University of Koblenz*, April 2005.

[10] S. Kagami, M. Mochimaru, Y. Ehara, N. Miyata, K. Nishiwak, T. Kanade, , H. Inoue, ”Measurement and comparison of humanoid H7 walking with human being ” ,*Robotics and Autonomous Sys*, vol. 48, pp. 177-187, 2003.

[11] L. Yang, C.M. Chew, T. Zielinska, A.N. Poo, “A Uniform Bipedal Gait Generator with Offline Optimization and Online Adjustable Parameters”, *Robotica 2007, Cambridge University Press*, United Kingdom, Vol. 25, pp. 549-565,2007.

[12] P. K. Schot, M. J. Decker, "The force driven harmonic oscillator model accurately predicts the preferred stride frequency for backward walking", *Human movement science 1998*, Vol. 17, pp. 67-76, 1995.

[13] K. Pepin, E Norman, H Barbeau, "Treadmill walking in incomplete spinal-cord-injured subjects: Adaptation to changes in speed" *Spinal Cord* , Vol. 41, pp. 257–270, 2003.

[14] D. E. Goldberg, ‘’Genetic Algorithms in Search, Optimiza- as a motion/force control problem’’, *J. Dyn. Syst. Meas.tion and Machine Learning, Addison-Wesley, Reading, Control Trans*. ASME, Vol. 114, pp. 660–665, 1992.