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# Total Discount Policy and Two Warehouses Strategy to Store Raw Materials with Economic Order Quantity Model

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**Abstract:** This study introduced an Economic Order Quantity (EOQ) model with payment in advance to purchase high-price raw materials. We relax and change some assumptions that were considered in earlier researches. At first we considered transportation cost as a linear function. Total discount policy is considered instead of incremental discount one. Also we developed model based on two warehouses strategy to store raw material in which holding cost is different for each of warehouses. We show that the model of this problem is shown to be a mixed-integer-nonlinear-programming type and in order to solve it, a simulated annealing approach is used. At the end, a numerical example is given to demonstrate the applicability of the proposed methodology in real world inventory control problems.

**Key words:** EOQ, joint replenishment, payment in advance, two warehouses, simulated annealing

## INTRODUCTION

Since its formulation in 1915, the square-root-formula for the Economic Order Quantity (EOQ) has been used in the inventory literature for a long time. This formula is based on the assumption of a constant demand. The discrete case of the dynamic version of EOQ was first discussed by Wagner and Whitin (1958). Regarding the continuous-time dynamic EOQ models, Silver and Meal (1969) were the first to suggest a simple modification of the classical square-root-formula in the case of time-varying demand. However, Donaldson (1977) discussed, for the first time, the classical no-shortage inventory policy for the case of a linear, time-dependent demand. His treatment was fully analytical and needed extensive computational effort to obtain the optimal solution.

The question of inventory shortages and backlogging were not considered at all by the aforementioned researchers. Deb and Chaudhuri (1987) were the first to incorporate shortages into the inventory lot-sizing problem with a linearly increasing time-varying demand. EOQ models for deteriorating items with a trended demand were considered by Goswami and Chaudhuri (1991, 1992), Xu and Wang (1990), Chung and Ting (1993, 1994), Kim (1995), Hariga (1995, 1996), Benkherouf (1995), Jalan *et al.* (1996), Jalan and Chaudhuri (1999), Giri and Chaudhuri (1997) and Lin *et al.* (2000), etc. In the model of Deb and Chaudhuri (1987), shortages are allowed in all cycles except the last one. Each of the cycles in which shortages are permitted starts with replenishment and ends with a shortage which is backlogged in the next cycle. Numerous studies have been carried out to address the problems of imperfect quality EMQ model with rework (Hayek and Salameh, 2001; Chiu, 2003; Chiu *et al.*, 2004; Jamal *et al.*, 2004). Chiu and Chiu (2006) studied optimal replenishment policy for an imperfect quality EMQ model with backlogging and failure in repair using conventional approach. That is to derive the optimal lot size by utilizing differential calculus on the expected cost function with the need to prove optimality first. Grubbstrom and Erdem (1999) first introduced an algebraic method to solve the classic EOQ and Economic Production Quantity (EPQ) model without the use of derivatives. A few researches have then been carried out using the same method (Cardenas-Barron, 2001; Wee and Chung, 2007). In these researches that extend the algebraic approach to an imperfect quality EMQ model examined by Chiu and Chiu (2006), the use of differential calculus is replaced with an algebraic method and the optimal lot size solution is obtained under the expected cost minimization.

Taleizadeh *et al.* (2008c) introduced an Economic Order Quantity (EOQ) model with payment in advance to purchase high-price raw materials. In this study we relax

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and change some of their assumptions. At first we considered transportation cost as a linear function. Total discount policy is considered instead of incremental discount one. Also we developed their model based on two warehouses strategy to store raw material in which holding cost is different for each of warehouses.

## PROBLEM DEFINITION

Inventory holding as a tactical-level decision against non-secure situations to increase confidence level of responding to customer demands and also to resist against all kind of non-deterministic situations in receiving raw materials made inventory control and management an important concept in supply chain management. The model that is proposed in this research is an applied model developed based on the real constraints and environments of manufacturing companies. In these companies, the annual demands of different products are first estimated at the start of the year. Then, based on these estimates, the inventory and planning departments proceed to material planning. In some cases, improper material planning and control policies and loss of sufficient raw materials in proper times and quantities, result to customer complaints and loss of customer and market share. In some instances gaining the lost-market-shares needs more promotion and advertising costs and causes increased production cost. To confront with these instances and to minimize raw-material shortages, in this research a model for material planning and control is developed.

In a manufacturing company, the steps involved in the ordering process of the materials are:

- Forecast the number of finished products
- Forecast the required raw-materials
- Order the required materials that consist of:
    - Releasing orders of the materials to a supplier
    - Paying a percentage ($\alpha$%) of the purchasing cost at $t_0$
    - Paying the remaining payment (1-$\alpha$)% at $t_{lc}$
    - Transportating materials and receiving them at T

Figure 1 shows the inventory control cycle of a material.

There are two payment methods in a material ordering process; (1) the credit transaction with a specific lead time and (2) cash method that has smaller lead time. In the cash method the purchasing cost of the total ordered material is paid to the supplier at the ordering time. However, in the credit transaction method $\alpha$% of the material purchasing cost is paid at the order release time ($t_0$) and the remaining (1-$\alpha$)% is paid at the starting time of the material transportation ($t_{lc}$). The ordered materials are received at T.

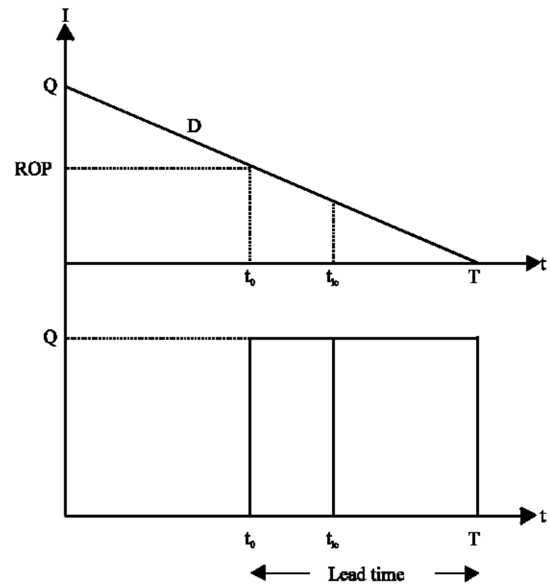


Fig. 1: Inventory control picture

Let us assume that the credit transaction method is used for payments in which the lead-time is deterministic and constant. Materials ordering and their transportation are done in batch form and the amount of raw materials in each batch and the number of batches in each vehicle is deterministic. Based on the product groups' consistency, the ordered materials can be carried together by finite-capacity vehicles with maximum capacity of F (in volume).

In this study, we plan to determine the optimal order quantity of each material in a joint replenishment policy such that the total cost is minimized. The costs are: (1) purchasing cost under total discount for each order, (2) holding cost for on hand inventory (including capital, warehouse and insurance costs), (3) capital cost of the next order, (4) transportation cost, (5) clearance cost and (6) fixed-order costs. Furthermore, we assume that the lead times are less than the cycle times of the materials.

All parameters of the problem are crisp and the quantity of the orders must be integer multiples of packets, each containing more than one product. Also we assume that the lead time is less than a cycle length. The constraints of the problem are space, budget and upper limit for the number of orders per year. Space constraint is different by common one incase if the total required space in each cycle exceeds than capacity of warehouse, the company should rent another warehouse.

## PROBLEM MODELING

**The parameters and the variables:** For i = 1,2,...,n and j = 1,2,...,P the parameters and the variables of the model are:

- $P$ = No. of the products
- $D_j$ = Annual constant demand of the jth product
- $n_j$ = No. of items in the packets of the jth product
- $h_1$ = Holding cost per unit of on hand inventory during a period in first warehouse (is same for all products)
- $h_1'$ = Holding cost per unit of on hand inventory during a period in rented warehouse (is same for all products)
- $h_{2j}$ = Capital cost per unit of the jth ordered product during the period before the payment of the remaining purchasing cost (10% of the total purchasing cost)
- $h_{3j}$ = Capital cost per unit of the jth ordered product during the period after the payment of the remaining purchasing cost
- $C_j^t$ = Transportation cost for each unit of the jth product
- $C_j^c$ = Clearance cost for each unit of the jth product
- $C_{ij}^p$ = Purchasing cost of the jth product in the ith discount break point
- $q_{ij}$ = ith discount break point of the jth product
- $ROP_j$ = Reorder point of the jth product
- $f_j$ = Space required for each packet of the jth product
- $L$ = Constant joint lead time for each order
- $A$ = Fixed order cost per each order
- $Q_j$ = Decision variable representing the order quantity of the jth product
- $m_j$ = Decision variable representing the number of packets that have been ordered for the jth product
- $TB$ = Total available budget
- $T$ = Decision variable representing the joint cycle length
- $N$ = No. of orders in each year (N = 1/T)
- $N_T$ = Upper limit for number of orders
- $C_H$ = Annual total holding cost of the products
- $C_c$ = Annual total clearance cost
- $C_P$ = Annual total purchasing cost of the products
- $C_T$ = Annual total transportation cost of the products
- $Z$ = Annual total costs

Here, the inventory model of the problem is developed.

**The objective function:** The objective function of the model is to minimize the total cost of the joint replenishment problem given in Eq. 1.

$$Z = C_T + C_P + C_C + C_A + C_H \tag{1}$$

The terms in the right hand side of Eq. 1 are derived as follows.

**Transportation cost ($C_T$):** The total transportation cost is calculated based on Eq. 2, in which $C_j^t$ is the transportation cost for each unit of the jth product and $Q_j$ is the number of order of jth product.

$$C_T = N\sum_{j=1}^{P} C_j^t Q_j = \frac{1}{T}\sum_{j=1}^{P} C_j^t D_j T = \sum_{j=1}^{P} C_j^t D_j \tag{2}$$

**Purchasing cost under total discount ($C_P$):** The purchasing cost of the company for the jth product at the beginning of a period can be calculated using the total discount policy. Let the total discount policy be as below in which $q_{ij}$ and $C_{ij}$; i = 1,2,...,n are the discount points and the purchasing costs for each unit of the jth product that corresponds to its lth discount break point, respectively.

$$C_j^P = \begin{cases} C_{1j}Q_j; & 0 < Q_j \le q_{1j} \\ C_{2j}Q_j; & q_{1j} < Q_j \le q_{2j} \\ \vdots & \\ C_{nj}Q_j; & Q_j \ge q_{nj} \end{cases} \tag{3}$$

In order to include the discount policy in the inventory model, we use Eq. 4 to model the total discount policy. Figure 2 shows this policy.

$$C_P = \sum_{j=1}^{P} C_{1j}\lambda_{1j}W_{1j} + C_{2j}\lambda_{2j}W_{2j} + \cdots + C_{nj}\lambda_{nj}W_{nj} = \sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}\lambda_{ij}W_{ij} \tag{4}$$

$$\begin{aligned} & 0 \le W_{1j} \le q_{1j}\lambda_{1j} \\ & q_{1j}\lambda_{2j} \le W_{2j} \le q_{2j}\lambda_{2j} \\ & \vdots \\ & q_{n-1,j}\lambda_{n-1,j} < W_{n-1,j} \le q_{nj}\lambda_{n-1,j} \\ & q_{nj}\lambda_{nj} < W_{nj} \\ & \lambda_{1j} + \lambda_{2j} + \cdots + \lambda_{nj} = 1 \\ & \lambda_{ij} = 0,1; \quad i = 1,2,\ldots,n \end{aligned} \tag{5}$$

**Clearance cost ($C_C$):** The clearance cost of the company for the jth product at each period is jth, the order quantity is $C_j^c$ and the number of order per year is $Q_j$. Hence, the total annual clearance cost of the company will be:

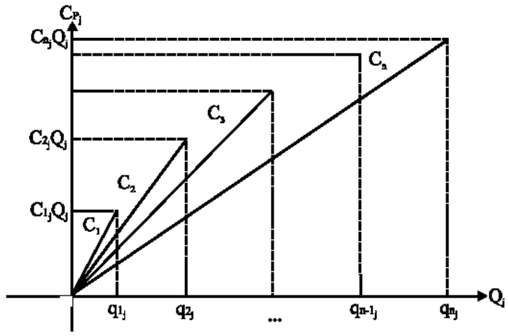

Fig. 2: Total discount policy to purchase the jth product

$$C_C = N\sum_{j=1}^{P} C_j^c Q_j = \sum_{j=1}^{P} C_j^c D_j \qquad (6)$$

**Fixed order cost ($C_A$):** The fixed order cost of each order per period is A and the number of orders per year is N. Hence and the total annual fixed order cost in a disjoint ordering policy will be NA. However, since we are using the joint replenishment policy, the fixed order cost will change to:

$$C_A = \frac{A}{T} \qquad (7)$$

**Holding cost ($C_H$):** The total available first warehouse space is F and the space required for each unit of the jth product is $f_j$. If the total required space for all products be more than F, the company should rent another warehouse. So the holding cost during a cycle will be:

$$\begin{cases} \sum_{j=1}^{P} h_1 Q_j; & \sum_{j=1}^{P} f_j Q_j \le F \\ h_1 \sum_{j=1}^{P} Q_j + h_1'[(\sum_{j=1}^{P} Q_j) - F]; & \sum_{j=1}^{P} f_j Q_j \ge F \end{cases} \qquad (8)$$

By introducing the binary variables $Y_1$, $Y_2$ Eq. 8 will change to Eq. 9 as below:

$$\begin{aligned} &C_h = h_1 W_1 + h_1' W_2 \\ &\sum_{j=1}^{P} Q_j = W_1 + W_2 \\ &FY_2 \le W_1 \le FY_1 \\ &0 \le W_2 \le MY_2 \\ &Y_1 \ge Y_2;\ \ Y_1, Y_2 = 0 \text{ or } 1,\ \ M \text{ is a very big number} \end{aligned} \qquad (9)$$

The first part of the capital cost occurs between the α% and (1-α)%- payment times and is derived as:

$$\sum_{j=1}^{P} h_j^2 Q_j (t_{1c} - t_0) \qquad (10)$$

The other part of the capital cost occurs between $[t_{1c}, T]$ and is calculated as:

$$\sum_{j=1}^{P} h_j^3 Q_j [L - (t_{1c} - t_0)] \qquad (11)$$

So, the total holding cost during a year will be:

$$\begin{aligned} &C_H = N\left[h_1 W_1 + h_1' W_2 + \sum_{j=1}^{P} h_j^2 Q_j (t_{1c} - t_0) + \sum_{j=1}^{P} h_j^3 Q_j [L - (t_{1c} - t_0)]\right] \\ &\sum_{j=1}^{P} Q_j = W_1 + W_2 \\ &FY_2 \le W_1 \le FY_1 \\ &0 \le W_2 \le MY_2 \\ &Y_1 \ge Y_2;\ \ Y_1, Y_2 = 0 \text{ or } 1,\ \ M \text{ is a very big number} \end{aligned} \qquad (12)$$

Also, according to (Fig. 1) we have:

$$Qj = D_j T_j \qquad (13)$$

Knowing that

$$N = \frac{1}{T} \rightarrow NT = 1 \qquad (14)$$

Equation 12 can be written as Eq. 14 for a joint order case:

$$\begin{aligned} &C_H = \frac{h_1 W_1 + h_1' W_2}{T} + \sum_{j=1}^{P} h_j^2 D_j (t_{1c} - t_0) + \sum_{j=1}^{P} h_j^3 Q_j [L - (t_{1c} - t_0)] \\ &\sum_{j=1}^{P} Q_j = W_1 + W_2 \\ &FY_2 \le W_1 \le FY_1 \\ &0 \le W_2 \le MY_2 \\ &Y_1 \ge Y_2;\ \ Y_1, Y_2 = 0 \text{ or } 1,\ \ M \text{ is a very big No.} \end{aligned} \qquad (15)$$

**The constraints:** Furthermore, since the total available budget is TB and the purchasing cost that is calculated is:

$$C_P = \sum_{j=1}^{P} \sum_{i=1}^{n} C_{ij} \lambda_{ij} W_{ij}$$

the budget constraint will be:

$$\frac{1}{T} \sum_{j=1}^{P} \sum_{i=1}^{n} C_{ij} \lambda_{ij} W_{ij} \le TB \qquad (16)$$

For the sake of convenience in ordering, clearance, transportation and some other activities, we assume an upper limit for the number of orders per year. In other words:

$$N \le N_T \rightarrow \frac{1}{T} \le N_T \rightarrow T \ge \frac{1}{N_T} \tag{17}$$

Moreover, the quantities of the orders must be integer multiples of packets, each containing more than one product. That is:

$$Q_j = n_j m_j \tag{18}$$

Knowing that:

$$Q_j = D_j T \tag{19}$$

We have:

$$TD_j = n_j m_j \tag{20}$$

Finally the model of the problem becomes:

$$
\begin{aligned}
\text{Min } Z &= C_T + C_P + C_C + C_A + C_H \\
&= \sum_{j=1}^{P} C_j^t D_j + \frac{1}{T}\sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}\lambda_{ij}W_{ij} + \sum_{j=1}^{P} C_j^c D_j + \frac{A}{T} + \frac{h_1 W_1 + h_1' W_2}{T} \\
&\quad + \sum_{j=1}^{P} h_j^2 D_j (t_{1c} - t_0) + \sum_{j=1}^{P} h_j^3 D_j [L - (t_{1c} - t_0)] \\
&= \frac{1}{T}\left[A + \sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}\lambda_{ij}W_{ij}\right] + \left[\frac{h_1 W_1 + h_1' W_2}{T}\right] \\
&\quad + \left[\sum_{j=1}^{P}(C_j^c + C_j^t)D_j + \sum_{j=1}^{P} h_j^2 D_j (t_{1c} - t_0) + \sum_{j=1}^{P} h_j^3 D_j [L - (t_{1c} - t_0)]\right] \\
\text{s.t}: \quad & \frac{1}{T}\sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}\lambda_{ij}W_{ij} \le TB \\
& \sum_{j=1}^{P} f_j Q_j \le F \\
& T \ge \frac{1}{N_T} \\
& TD_j = n_j m_j \\
& 0 \le W_{1j} \le q_{1j}\lambda_{1j} \\
& q_{1j}\lambda_{2j} \le W_{2j} \le q_{2j}\lambda_{2j} \\
& \qquad \vdots \\
& q_{n-1,j}\lambda_{n-1,j} < W_{n-1,j} \le q_{nj}\lambda_{n-1,j} \\
& \qquad q_{nj}\lambda_{nj} < W_{nj} \\
& \lambda_{1j} + \lambda_{2j} + \cdots + \lambda_{nj} = 1 \\
& \lambda_{ij} = 0,1;\ \forall j,\ j = 1,2,\cdots,P\ ,\forall i,\ i = 1,2,\cdots,n \\
& \sum_{j=1}^{P} Q_j = W_1 + W_2 \\
& FY_2 \le W_1 \le FY_1 \\
& 0 \le W_2 \le MY_2 \\
& Y_1 \ge Y_2; \quad Y_1, Y_2 = 0 \text{ or } 1, \quad M \text{ is a very big number} \\
& T \ge 0, m_j, Q_j \ge 0 \ \text{integer}
\end{aligned}
\tag{21}
$$

## A SOLUTION ALGORITHM

Since the model in Eq. 21 is mixed-integer-nonlinear in nature, reaching an analytical solution (if any) to the problem is difficult (Gen and Cheng, 1997). Hence, we need to employ a meta-heuristic search algorithm to solve it. Furthermore, as efficient treatment of mixed integer nonlinear optimization is one of the most difficult problems in practical optimization (El-Sharkawi, 2008), we need to employ a meta-heuristic search algorithm to solve it.

New ways have been found to optimize problems for less than a century, but nature has used various ways of optimization for millions of million years. Recently scientists mimicked nature to solve different kinds of complex optimization problems. Most of these problems are so complicated and time consuming that we cannot use an exact algorithm to solve them. Thus, typically some non-precise algorithms are used to find a near optimum solution in a shorter period. We call these algorithms meta-heuristic (Dorigo and Stutzle, 2004). Many researchers have successfully used meta-heuristic methods to solve complicated optimization problems in different fields of scientific and engineering disciplines. Some of these meta-heuristic algorithms are simulating annealing (Aarts and Korst, 1989; Taleizadeh *et al*., 2008a), threshold accepting (Dueck and Scheuer, 1990), Tabu search (Joo and Bong, 1996), genetic algorithms (Al-Tabtabai and Alex, 1999; Taleizadeh *et al*., 2008b), neural networks (Gaiduk *et al*., 2002), ant colony optimization (Dorigo and Stutzle, 2004), fuzzy simulation (Taleizadeh *et al*., 2009), evolutionary algorithm (Laumanns *et al*., 2002) and harmony search (Lee and Geem, 2004; Geem *et al*., 2001; Taleizadeh *et al*., 2008c).

Simulated Annealing (SA) is a local search method that finds its inspiration in the physical annealing process studied in statistical mechanics (Aarts and Korst, 1989). It is an effective and efficient method that produces good suboptimal solutions and has diffused rapidly into many combinatorial optimization areas (Kirkpatrick *et al*., 1994). An SA algorithm repeats an iterative neighbor generation procedure and follows search directions that improve the objective function value. While exploring solution space, the SA method offers the possibility of accepting worse neighbor solutions in a controlled manner in order to escape from local minima. More precisely, in each iteration, for a current solution x characterized by an objective function value f(x'), a neighbor x' is selected from the neighborhood of x denoted N(x) and defined as the set of all its immediate neighbors. For each move, the objective difference Δ = f(x')-f(x) is evaluated. For minimization problems, x' replaces x whenever $\Delta \leq 0$. Otherwise, x' could also be accepted with a probability

$$P = e^{\frac{-\Delta}{T}}$$

The acceptance probability is compared to a number RN ϵ Uni(0,1) generated randomly and x' is accepted whenever P>RN.

The factors that influence the acceptance probability are the degree of objective function value degradation $\Delta$ (smaller degradations induce greater acceptance probabilities) and the parameter T called temperature (higher values of T give higher acceptance probability). The temperature can be controlled by a cooling scheme specifying how it should be progressively reduced to make the procedure more selective as the search progresses to neighborhoods of good solutions. There exist theoretical schedules, which require infinite computing time guaranteeing asymptotic convergence toward the optimal solution. In practice, much simpler and finite computing time schedules are preferred even if they do not guarantee an optimal solution.

A typical finite time implementation of SA consists of decreasing the temperature T in S steps, starting from an initial value $T_0$ and using an attenuation factor $\alpha$, $(0<\alpha<1)$. The initial temperature $T_0$ is supposed to be high enough to allow acceptance of any new neighbor proposed in the first step. In each step s, the procedure generates a fixed number of neighbor solutions $N_{sol}$ and evaluates them using the current temperature value $Ts = \alpha^s T_0$. The whole process is commonly called cooling chain or also Markov chain. Adaptation of SA to an optimization problem consists in defining its specific components: a neighbor generation procedure, objective function evaluation, a method for assigning the initial temperature, a procedure to change the temperature and a cooling scheme including stopping criteria. These adaptation steps for our new adaptations of SA for the inventory model of Eq. 21 are described here.

**A neighbor generation procedure:** The neighbor generation is an important component of SA. It has to be designed to allow easy neighbor generation and fast calculation of $\Delta$ and guarantee accessibility for the entire solution space.

In this study, the initial solutions are generated in two groups. In the first group, the procedure randomly chooses the initial solution from the possible solution space. For the other groups of initial solutions the procedure employs some solutions of a Genetic algorithm described here.

**Evaluating the objective function:** After creating each solution, the objective function should be evaluated to compare the jth and the ith solutions. In a maximization problem, if the objective function of the new solution (j) is bigger than the previous solution (i), then (j) will be accepted. Otherwise, by generating a random number the better solution will be selected.

**Assigning the initial temperature:** Temperature is one of the important parameters of the SA algorithm that affects the acceptance or non-acceptance of the objective function changes. The initial temperature should be chosen such that at the first level a big number of inappropriate solutions are accepted. In this way, more changes are possible and hence more solutions are explored. Furthermore, the time required for the process repetition along with the annealing process depends on the initial temperature. In this research, the large values of 1000, 2000 and 5000 are chosen for initial temperatures.

**Changing the temperature:** One of the primary aspects of the annealing process in a SA algorithm is the range of temperature change. The temperature plays an important role in the possibility of selecting a bad solution. On one hand, when the temperature assumes a high value, a big number of bad solutions are accepted, leading to selecting a local optimum point. On the other hand, when the temperature is low, the probability of the solution to be a local optimum is high. In this study, we change the temperature of the SA algorithm based on a geometric function given in Eq. 22 with $\alpha$ = 0.9, 0.95 and 0.99.

$$Ts = \alpha T_{s-1};\ s = 1,2,\ldots \text{ and } 0<\alpha<1 \tag{22}$$

**Cooling scheme and stopping criterion:** In a specific temperature of a SA algorithm, it is necessary to analyze the equilibrium state after a couple of renitence to see if the annealing process will be continued in that temperature or it will be stopped and moved to the next temperature. An epoch is the number changes that should occur in a specific temperature.

Different types of stopping criteria have been presented for different SA algorithms in the literature. Some of them are:

- A lower bound on the final temperature
- An upper bound on the total number of states
- Reaching a solidification point based on the value of a function evaluated on the value of the objective function and the temperature

- An upper bound on the total number of exchanges accepted along the annealing process
- An upper bound on the total number of refused changes

In this research, we select the first criterion and stop the algorithm after it reaches the final temperature $T_F$. Furthermore, we use 100, 200 and 500 for different values of N(t).

In short, the steps involved in the SA algorithm used in this research are:

1 Choosing an initial solution i from the group of the feasible solutions S
2 Choosing the initial temperature $T_0>0$
3 Selecting the number of iterations N(t) at each temperature
4 Selecting the final temperature $T_F$
5 Determining the process of the temperature reduction until it reaches $T_F$
6 Setting the temperature exchange counter n to zero for each temperature. (Balancing process)
7 Creating the j solution at the neighborhood of the i solution
8 Evaluating the objective function f = Z at any temperature.
9 Calculating $\Delta = f(j)-f(i)$
10 Accepting the solution j, if $\Delta<0$. Otherwise, generating a random number RN~Uniform (0,1). If

$$RN < e^{\left(\frac{-\Delta}{T_0}\right)}$$

then selecting the j solution
11 Setting n = n+1. If n is equal to N(t) then go to 12. Otherwise, go to 7
12 Reducing the temperature. If it reaches $T_F$ then stop. Otherwise, go to 6

## NUMERICAL EXAMPLE

Consider a multi-product inventory control problem with twenty products and general data given in Table 1. The total available warehouse space for first store, the total budget and the maximum number of order are F = 500, TB = 170,000,000 and $N_T = 6$, respectively. Also, L = 2.5 months, A = 2000, $(t_0) = 0$, $h_1 = 40$, $h_1' = 50$ and $(t_{1c}) = 1.5$ months. Table 2 shows different values of the SA parameters used to obtain the solution. In this research, all of the possible combinations of the SA parameters are employed and using the min criterion the best combination of the parameters has been selected. Table 3 shows the best combination of the SA algorithm and the best result is shown in Table 4. Furthermore, the convergence path of the best result of the objective function is shown in Fig. 3.

Table 1: General data of numerical example

| Product | $D_j$ | $n_j$ | $h_{2j}$ | $h_{3j}$ | $C^t_j$ | $C^c_j$ | $C^p_{1j}$ | $C^p_{2j}$ | $C^p_{3j}$ | $C^p_{4j}$ | $q_{1j}$ | $q_{2j}$ | $q_{3j}$ | $f_j$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1000000 | 20000 | 3 | 30 | 4 | 75 | 70 | 65 | 60 | 55 | 150000 | 250000 | 350000 | 2 |
| 2 | 950000 | 15000 | 3 | 30 | 5 | 75 | 70 | 65 | 60 | 55 | 150000 | 250000 | 350000 | 3 |
| 3 | 900000 | 10000 | 3 | 30 | 6 | 75 | 70 | 65 | 60 | 55 | 150000 | 250000 | 350000 | 2 |
| 4 | 850000 | 5000 | 3 | 30 | 4 | 75 | 70 | 65 | 60 | 55 | 150000 | 250000 | 350000 | 3 |
| 5 | 800000 | 20000 | 3 | 30 | 5 | 75 | 70 | 65 | 60 | 55 | 150000 | 250000 | 350000 | 2 |
| 6 | 750000 | 15000 | 4 | 40 | 6 | 100 | 95 | 90 | 85 | 80 | 75000 | 125000 | 200000 | 3 |
| 7 | 700000 | 10000 | 4 | 40 | 4 | 100 | 95 | 90 | 85 | 80 | 75000 | 125000 | 200000 | 2 |
| 8 | 650000 | 5000 | 4 | 40 | 5 | 100 | 95 | 90 | 85 | 80 | 75000 | 125000 | 200000 | 3 |
| 9 | 600000 | 20000 | 4 | 40 | 6 | 100 | 95 | 90 | 85 | 80 | 75000 | 125000 | 200000 | 2 |
| 10 | 550000 | 15000 | 4 | 40 | 4 | 100 | 95 | 90 | 85 | 80 | 75000 | 125000 | 200000 | 3 |
| 11 | 500000 | 10000 | 5 | 50 | 5 | 145 | 140 | 135 | 130 | 125 | 30000 | 60000 | 90000 | 2 |
| 12 | 450000 | 5000 | 5 | 50 | 6 | 145 | 140 | 135 | 130 | 125 | 30000 | 60000 | 90000 | 3 |
| 13 | 400000 | 20000 | 5 | 50 | 4 | 145 | 140 | 135 | 130 | 125 | 30000 | 60000 | 90000 | 2 |
| 14 | 350000 | 15000 | 5 | 50 | 5 | 145 | 140 | 135 | 130 | 125 | 30000 | 60000 | 90000 | 3 |
| 15 | 300000 | 10000 | 5 | 50 | 6 | 145 | 140 | 135 | 130 | 125 | 30000 | 60000 | 90000 | 2 |
| 16 | 250000 | 5000 | 6 | 60 | 4 | 200 | 195 | 190 | 185 | 180 | 10000 | 40000 | 70000 | 3 |
| 17 | 200000 | 20000 | 6 | 60 | 5 | 200 | 195 | 190 | 185 | 180 | 10000 | 40000 | 70000 | 2 |
| 18 | 150000 | 15000 | 6 | 60 | 6 | 200 | 195 | 190 | 185 | 180 | 10000 | 40000 | 70000 | 3 |
| 19 | 100000 | 10000 | 6 | 60 | 4 | 200 | 195 | 190 | 185 | 180 | 10000 | 40000 | 70000 | 2 |
| 20 | 50000 | 5000 | 6 | 60 | 5 | 200 | 195 | 190 | 185 | 180 | 10000 | 40000 | 70000 | 3 |

Table 2: The parameters of the SA algorithm

| N (t) | $T_0$ | α |
|---|---|---|
| 100 | 1000 | 0.90 |
| 200 | 2000 | 0.95 |
| 500 | 5000 | 0.99 |

Table 3: The best combination of the SA parameters

| α | $T_0$ | N (t) |
|---|---|---|
| 0.95 | 2000 | 500 |

Table 4: The best result

| Product | T | Z | $m_j$ | $Q_j$ |
|---|---|---|---|---|
| 1 | 0.1671 of year | 129,310,000 | 9 | 180000 |
| 2 | | | 11 | 165000 |
| 3 | | | 16 | 160000 |
| 4 | | | 29 | 145000 |
| 5 | | | 7 | 140000 |
| 6 | | | 9 | 135000 |
| 7 | | | 12 | 120000 |
| 8 | | | 22 | 110000 |
| 9 | | | 6 | 120000 |
| 10 | | | 7 | 105000 |
| 11 | | | 9 | 90000 |
| 12 | | | 16 | 80000 |
| 13 | | | 4 | 80000 |
| 14 | | | 4 | 60000 |
| 15 | | | 6 | 60000 |
| 16 | | | 9 | 45000 |
| 17 | | | 2 | 40000 |
| 18 | | | 2 | 30000 |
| 19 | | | 2 | 20000 |
| 20 | | | 2 | 10000 |

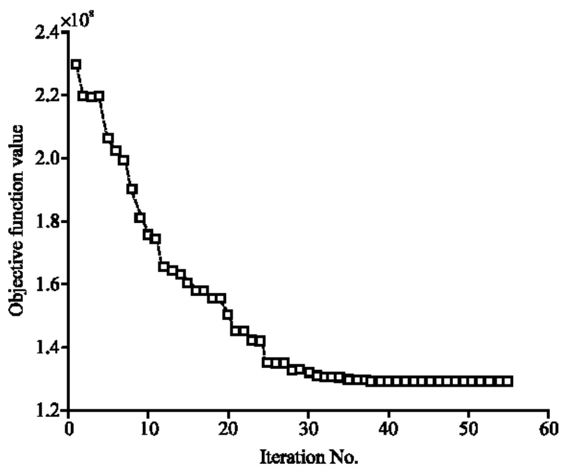


Fig. 3: Convergence graph of SA algorithm

## CONCLUSION AND RECOMMENDATIONS FOR FUTURE RESEARCH

In this study, a joint replenishment multi-product multi constraint inventory model with two warehouse space with different holding cost based on EOQ model to purchase high price raw materials was investigated. A mathematical modeling by total discount, clearance, fixed order, holding, shortage and transportation costs was introduced and shown to be integer-nonlinear programming problems. Then, a meta-heuristic algorithm (Simulated Annealing) has been proposed to solve the integer non-linear problem.

Some recommendations for future works follow:

- Shortage can be included in the model
- Stochastic demands or lead time can be taken into account
- It can be considered that lead time may exceed than cycle length
- Deterioration rate for inventory in the warehouse can be considered

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