NEUBER'S RULE: A NUMERICAL ANALYSIS

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ABSTRACT

The history of deformation at critical points is of paramount importance in low cycle fatigue. In parts with sudden changes in geometry, due to the stress concentration effect plasticity may occur close to the notches even under nominal stress much lower than the yield strength of the material. This paper aims to analyze the stress and strain state near the notches using the Neuber rule and the Ramberg-Osgood description of the stress/strain relationship.

MATLAB software routines were developed to analyze theoretical results, and these results will be compared with finite element method results using Abaqus software.

In order to evaluate Neuber 's rule both analytically and numerically, a plate with a central circular hole, subjected to monotonic and cyclic loading, was analyzed to evaluate the behavior near the notch. Two different materials were considered. One of the materials has no hardening, i.e., elastic perfectly plastic material; the other material was the S355 steel, displaying substantial hardening.

While the stress concentration factor K_σ and the strain concentration factor K_ε have the same value in elastic regime, in the elasto-plastic regime the stress concentration factor K_σ tends to decrease and the strain concentration factor K_ε tends to increase with increase in plastic deformation.

For the cases analyzed, Neuber's rule gives accurate results for stresses and deformations near notches for $\sigma/\sigma y<0.6$. The errors for stress and strain resulting from the use of Neuber's rule, taking as reference the non-linear Abaqus analyses, are discussed and presented in this article.

KEYWORDS: Neuber rule; Ramberg-Osgood; stress concentration factor; strain concentration factor; Abaqus

INTRODUCTION

The history of deformation at critical points is of paramount importance in low cycle fatigue. In parts with sudden changes in geometry, due to the stress concentration effect, plasticity may occur close to the notches even under nominal stress much lower than the yield strength of the material. This paper aims to analyze the stress and strain state near the notches using the Neuber's rule and the Ramberg-Osgood description of the stress/strain relationship. MATLAB software routines were developed to analyze theoretical results, and these results were compared with finite element method results using Abaqus software.

In order to evaluate Neuber's rule both analytically and numerically, a plate with a central circular hole, subjected to monotonic and cyclic loading, was analyzed to evaluate the behavior near the notch. Two different materials were considered. One of the materials has no hardening, i.e., elastic perfectly plastic material; the other material was the S355 steel, displaying substantial hardening.

RAMBERG-OSGOOD

Ramberg-Osgood equation aims to evaluate the non-linear relationship between stress and strain in plastic regime. In other words, Ramberg-Osgood is useful to model material hardening. Deformations can be divided in elastic and plastic deformation. Elastic deformation is the one that once the load is removed the deformation disappear while the plastic deformation remains. According to Ramberg-Osgood the relation between total deformation, elastic and plastic deformations can analyzed from equation (1).

$$\varepsilon = \varepsilon_e + \varepsilon_P = \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^{\frac{1}{n}} \quad (1)$$

NEUBER'S RULE

In uniaxial linear elastic case Neuber verified that $K_t = K_\varepsilon = K_\sigma$. After the material's yield stress, the relation between stress/strain is no longer linear. On plastic regime for elastic-perfectly plastic materials equations (2) is used.

$$K_\sigma . \sigma_n = \sigma_Y ;\ K_t > K_\sigma \quad (2)$$

It becomes important to highlight that the effect of the stress concentration factor during fracture depends on material behaviour. In ductile materials will occur deformation on notches decreasing the value of the stress concentration factor while fragile materials will break. High resistance materials are harder to deform next to notches, thus the stress concentration factors will remain high while in low resistance materials deformation will occur. The notches on low resistance materials will become less sharp decreasing the stress concentration. Further explanations can be found, for e.g., in de Castro and Meggliolaro [1].

In ductile parts is possible to analyze that the relation between $\frac{K_\sigma}{K_t}$ tends to decrease while the relation between $\frac{K_\varepsilon}{K_t}$ tends to increase.

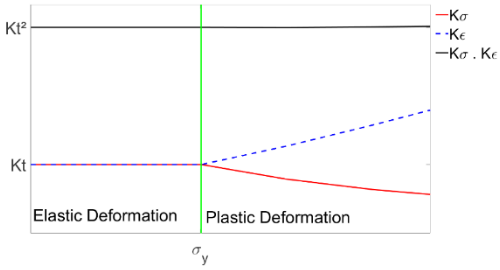

Figure 1. Schematic figure of K_σ, K_ε and product of $K_\sigma . K_\varepsilon$ behaviour.

Neuber, while investigating strain concentration in plastic regime in prismatic parts of non-linear material under torsion and low deformation, proved that:

$$K_t^2 = \frac{\sigma.\varepsilon}{\sigma_n.\varepsilon_n} \tag{3}$$

It became common to generalize the effect of stress concentration to any kind of part or load under plane stress. This relation became known as Neuber's Rule.

3.1. Monotonic Loading

According to previously shown equations becomes possible to evaluate stress and strains next to notches as function of nominal stress. Combining equations (1) and (3) is possible to obtain the relation between nominal and real stress under monotonic loading

$$K_t^2.\sigma_n\left(\frac{\sigma_n}{E} + \left(\frac{\sigma_n}{K}\right)^{\frac{1}{n}}\right) = \sigma\left(\frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^{\frac{1}{n}}\right) \tag{4}$$

Since the left-hand side of the equation is known is possible to calculate local stress. Strains for monotonic loading can be evaluated from equation (1).

3.2. Cyclic Loading

In cyclic loading cases turns out to be necessary to use nominal stress amplitude $\Delta\sigma_n$ instead of nominal stress. Ramberg-Osgood equation will become:

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K}\right)^{\frac{1}{n}} \tag{5}$$

And Neuber's Rule for cyclic loading will become:

$$K_t^2 = \frac{\Delta\sigma.\Delta\varepsilon}{\Delta\sigma_n.\Delta\varepsilon_n} \tag{6}$$

Combining equations (5) and (6) makes possible to obtain an equation that relates nominal stress and stress and strain next to notches.

$$K_t^2.\Delta\sigma_n\left(\frac{\Delta\sigma_n}{E} + \left(\frac{\Delta\sigma_n}{K}\right)^{\frac{1}{n}}\right) = \Delta\sigma\left(\frac{\Delta\sigma}{E} + \left(\frac{\Delta\sigma}{K}\right)^{\frac{1}{n}}\right) \tag{7}$$

ANALYZED MODEL

In order to analyze Neuber's rule numerically and analytically was developed a finite element model of a plate with a hole loaded remotely. The plate is squared with dimensions 100 mm and a center hole with radius of 10mm. The plate will be submitted to different monotonic loadings of up to 350 MPa. It will also be submitted to a cyclic loading in order to analyze the behaviour next to the notch.

Two different materials were studied. One of them will have no hardening and will be considered as an elastic perfectly plastic material, EPP. The other will be steel S355 with great hardening. Mechanical properties can be analyzed on table 1.

Table 1. Mechanical Properties of elastic perfectly plastic material and Steel S355

	Elastic Perfectly Plastic	S355
Young's Modulus (GPa)	200	207
Yield Strength (MPa)	400	460

The plate geometry was modelled on software Abaqus in a 2D model of 1/4 of the plate. Symmetry condition was applied in axis X and Y. The loading was applied as a pressure. A structured mesh with refining next to the notch was used. Quadratic elements with reduced integration and plane stress CPS8R were applied to the model. Boundary conditions and mesh can be analyzed from figure 2.

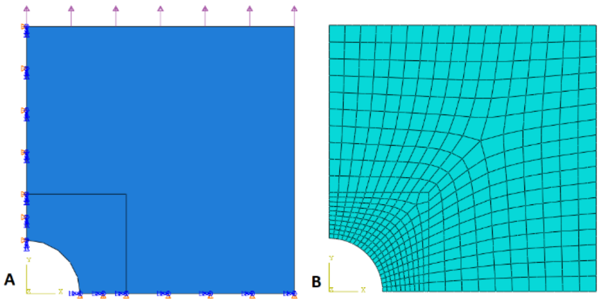

Figure 2. (A) Boundary conditions applied to the model; (B) Mesh used in Abaqus.

4.1. Evaluation of K_t

The proposed geometry has a stress concentration factor K_t=3. In order to estimate K_t numerically in elastic regime was applied a 100 MPa load to EPP material. The 100 MPa load was chosen because theoretically should not have any plastic strain, thus $K_t=K_\sigma=K_\varepsilon$. Figure 3 shows that the maximum stress is 299.9 MPa on the hole border. The K_t evaluated as 3 has an error of 0.033%.

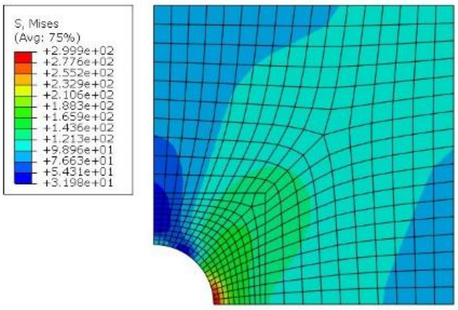

Figure 3. Numerical results for material EPP and load of 100 MPa.

3.2. Ramberg-Osgood Modelling

In order to model material hardening Ramberg-Osgood was used. Parameters for material S355 were extracted from a work made by Jesus et al. [2]. Figure 5 shows the comparison between experimental and Ramberg-Osgood results. Parameters K and n can be evaluated from table 2.

Table 2. Parameter used in Ramberg-Osgood model

	Elastic Perfectly Plastic	S355
K	401	595.85
n	0.001	0.0757

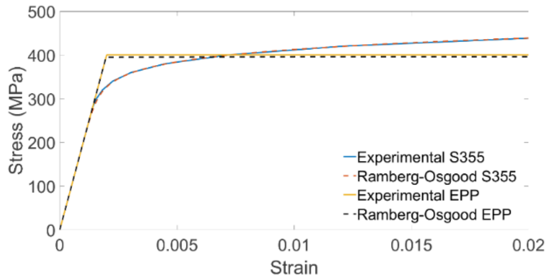

Figure 4. Comparison between experimental and Ramberg-Osgood modelling.

It is important to highlight that the results are so precise that in part of the domain the curves superimpose each other.

MONOTONIC LOADING RESULTS

5.1. Material Elastic Perfectly Plastic

Figure 6 shows the graphic results of Neuber's rule for different loadings applied on EPP material obtained in software MATLAB. Tables 3 and 4 shows the comparison between results obtained from ABAQUS and MATLAB and the error.

Table 3. Overview of stress results for ABAQUS simulations and Neuber's rule on MATLAB routine for material EPP

Load (MPa)	Abaqus stress (MPa)	Neuber's rule stress (MPa)	Error (%)
100	299.9	300	0.033
150	400.93	400	0.23
200	400.70	400	0.175
250	400.72	400	0.18
300	400.74	400	0.185

Table 4. Overview of strain results for ABAQUS simulations and Neuber's rule on MATLAB routine for material EPP

Load (MPa)	Abaqus strain	Neuber's rule strain	Error (%)
100	0.0015	0.0015	0
150	0.00283	0.00254	10.24
200	0.00458	0.0045	1.74
250	0.00818	0.00703	14.06
300	0.0184	0.01013	44.94

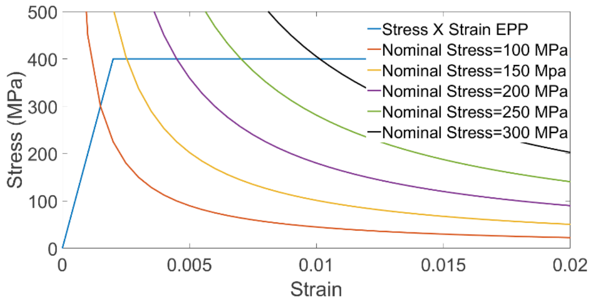

Figure 5. Graphic analysis for stress and strain for Neuber's Rule. Material EPP

Figure 6 shows values for K_ε, K_σ and the product $K_\varepsilon.K_\sigma$ in function of normalized yield strength. It is possible to see that the product of $K_\varepsilon.K_\sigma$ is constant up to $\frac{\sigma}{\sigma_Y} \leq 0.6$. For values of $\frac{\sigma}{\sigma_Y} > 0.6$ the product of $K_\varepsilon.K_\sigma$ explodes. In other words, Neuber's rule seems not to be valid for $\frac{\sigma}{\sigma_Y} > 0.6$.

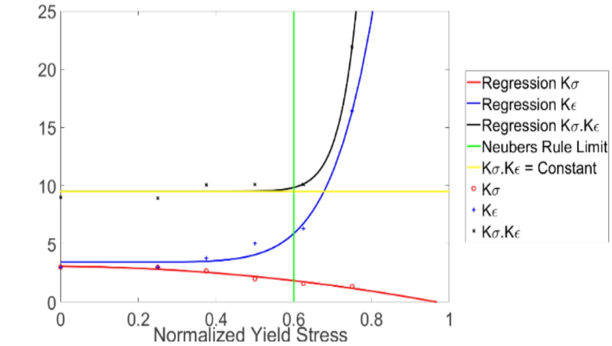

Figure 6. Graphic analysis for stress and strain for Neuber's Rule. Material EPP

5.2. Material S355

Figure 7 shows the graphic results of Neuber's rule for different loadings applied on steel S355 obtained in software MATLAB. Table 5 and 6 shows the comparison between results obtained from ABAQUS and MATLAB and the error.

Table 5. Overview of stress results for ABAQUS simulations and Neuber's rule on MATLAB routine for material S355

Load (MPa)	Abaqus stress (MPa)	Neuber's rule stress (MPa)	Error (%)
100	267.3	291	8.98
150	324.2	354	9.25
200	380.6	381	0.26
250	426.9	398	6.57
300	460.9	414	10.0
350	519.1	436	15.9

Table 6. Overview of strain results for ABAQUS simulations and Neuber's rule on MATLAB routine for material S355

Load (MPa)	Abaqus strain	Neuber's rule strain	Error (%)
100	0.001792	0.001499	16.35
150	0.003437	0.00276	19.69
200	0.006351	0.00456	28.20
250	0.01679	0.00687	59.08
300	0.0372	0.0102	72.58
350	0.166	0.0185	88.85

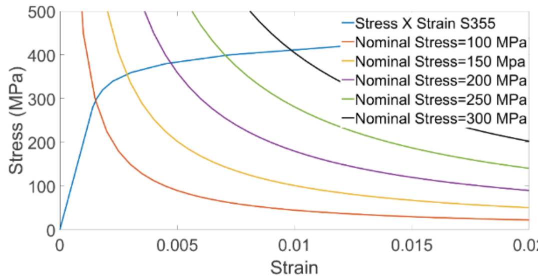

Figure 7. Graphic analysis for stress and strain for Neuber's Rule. Material EPP

Figure 8 shows values for K_ε, K_σ and the product $K_\varepsilon.K_\sigma$ as a function of normalized yield strength. It is possible to see that the product of $K_\varepsilon.K_\sigma$ is constant up to $\frac{\sigma}{\sigma_Y} \leq 0.9$. For values of $\frac{\sigma}{\sigma_Y} \leq 0.9$ the product of $K_\varepsilon.K_\sigma$. In other words, Neuber's rule seems not to be valid for $\frac{\sigma}{\sigma_Y} \leq 0.9$.

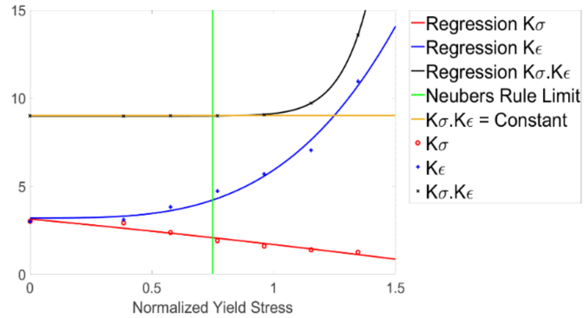

Figure 8. Graphic analysis for stress and strain for Neuber's Rule. Material EPP

CYCLIC LOADING RESULTS

In this section will be presented the results to cyclic loading applied to the plate. For material EPP the applied load block was (0 →200 →-200 → 200 →-200 →200) MPa and for S355 the applied loading block was (0 →150 →-150 → 150 →-150 → 150) MPa. Both cases were analyzed under R=-1. The loading magnitudes were selected in order to be lower than 60% of the materials yield stress.

Table 7 shows the results for Δε and Δσ and its errors for material EPP. The obtained results were analyzed in function of Δε and Δσ since those are the main driving forces of low cycle fatigue.

Table 7. Comparison between hysteresis loop for material elastic perfectly plastic

	Abaqus	Neuber's rule	Error (%)
Δε	0.009999	0.009	9.90
Δσ (MPa)	873	800	8.36

Figures 9 and 10 shows hysteresis loop for load blocks applied to both materials.

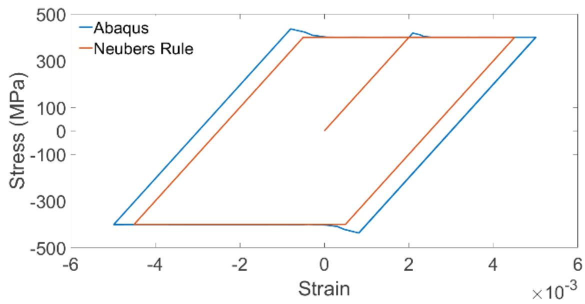

Figure 9. Comparison between Stress-Strain obtained from Abaqus and Neuber's rule for material EPP

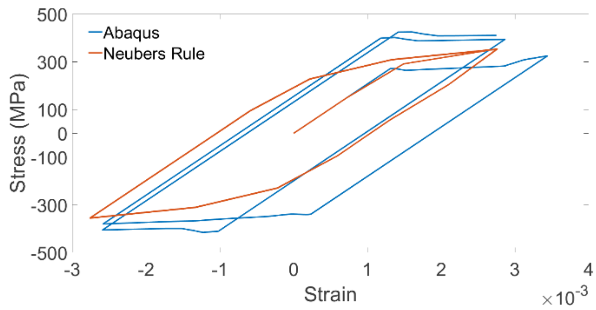

Figure 10. Comparison between Stress-Strain obtained from Abaqus and Neuber's rule for material S355

It is important to highlight that the hysteresis loop obtained through Abaqus shows material hardening. This phenomenon can be observed through the fact that besides same stress range applied the strains on the second loop will become smaller.

Since the material behaviour changes from one loop to another the errors will be calculated to each loop individually. The results shown on table 6 show good convergence with those obtained through Neuber's Rule and Ramberg-Osgood.

Table 8. Comparison between hysteresis loop 1 for material S355

	Abaqus	Neuber's rule	Error (%)
Δε	0.005448	0.00552	1.32
Δσ (MPa)	774	709.82	8.29

Table 9. Comparison between hysteresis loop 2 for material S355

	Abaqus	Neuber's rule	Error (%)
Δε	0.005336	0.00552	3.45
Δσ (MPa)	815	709.82	12.9

CONCLUSIONS

From the results obtained in this work is possible to reach to some conclusions. The stress concentration factor K_σ and strain concentration factor K_ε have the same value in elastic regime. When plasticity occurs the stress concentration factor tends to decrease while the strain concentration factor tends to increase.

Neuber's rule showed good accuracy for loads lower than 60% of the materials yield stress. For loads $\frac{\sigma}{\sigma_Y} > 0.6$ on monotonic loadings Neuber's rule leads to wrong calculations.

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