

# **Um sobrevoo do problema da fadiga**

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4200-465 Porto  
Portugal

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## **Introdução – exs. de casos**

- Haste
- Ligação soldada

## **Referência a conceitos básicos**

- Bibliografia de autores do DEMec da FEUP

## **Propagação de fendas**

- Expansão de furos

## **Propagação de fendas em modo misto**

- O caso da flexão em 4 pontos

## **Métodos numéricos – o XFEM**

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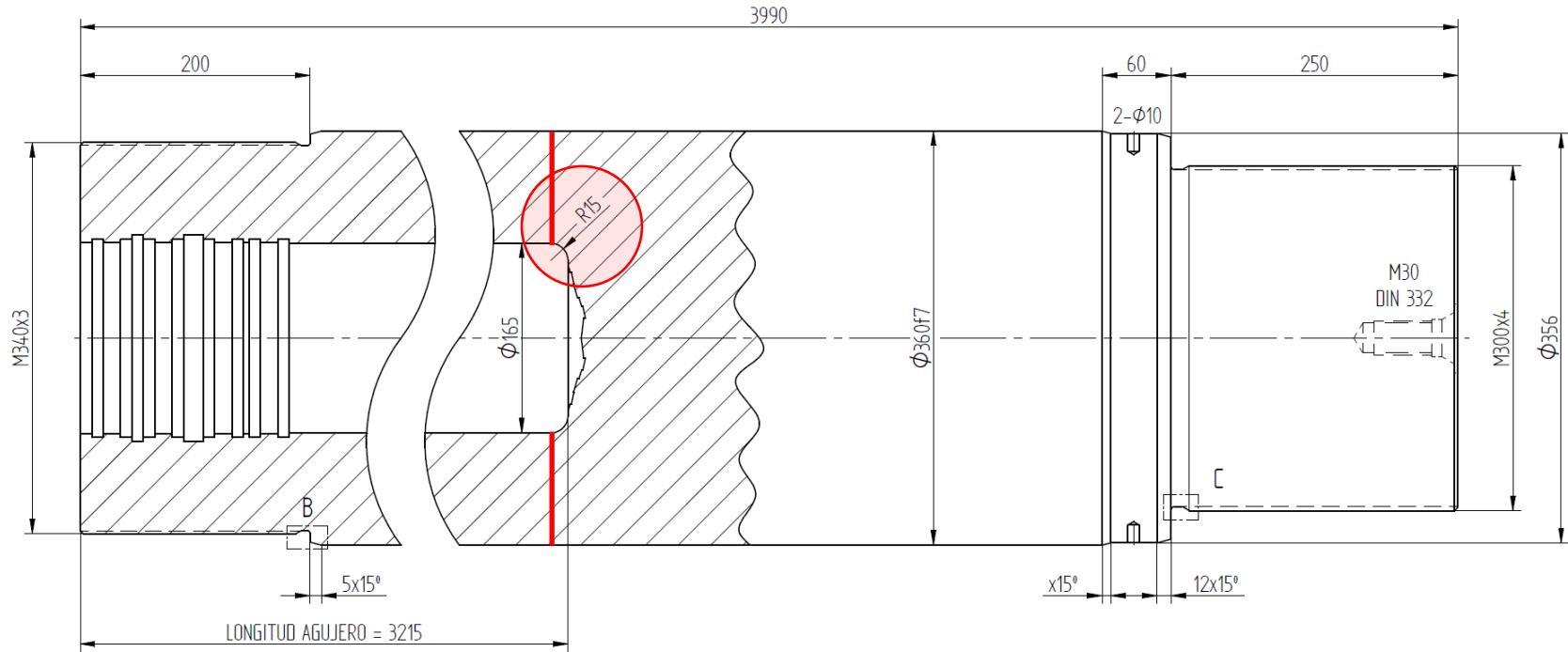
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## technical drawing and fracture location

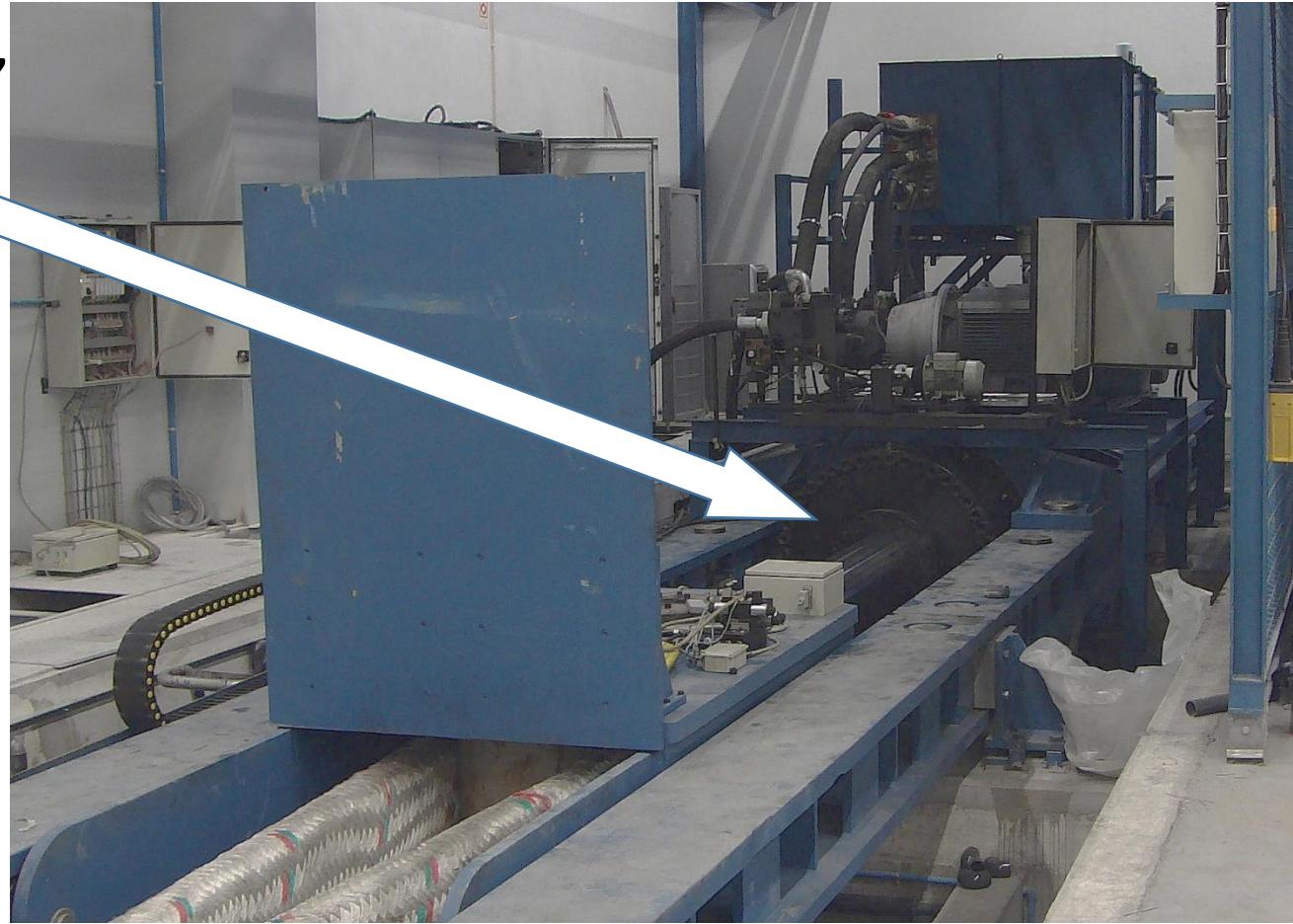


S.M.O. Tavares, N. Viriato, M. Vaz, P.M.S.T. de Castro, 'Failure analysis of the rod of a hydraulic cylinder', *Procedia Structural Integrity*, vol.1, pp.173-180, 2016

## fatigue testing machine for large cables for marine applications

Lankhorst, Maia, Portugal; >1500 t capacity

4m long rod,  
ext. diam.  
340mm

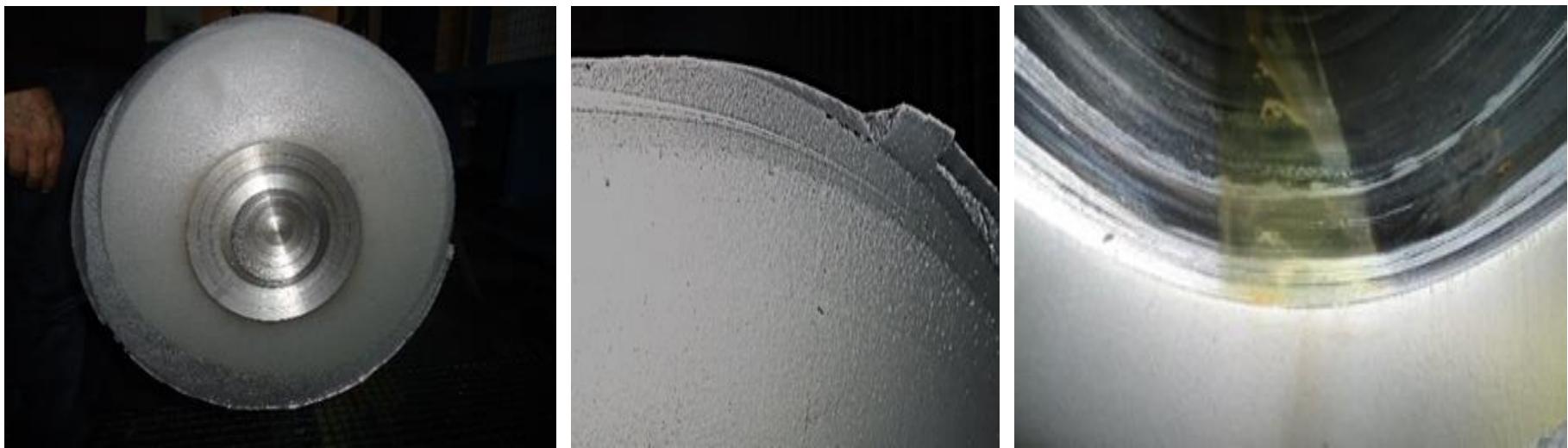


another example of high capacity fatigue testing

TWI, Abington, UK; max. load ~600 t

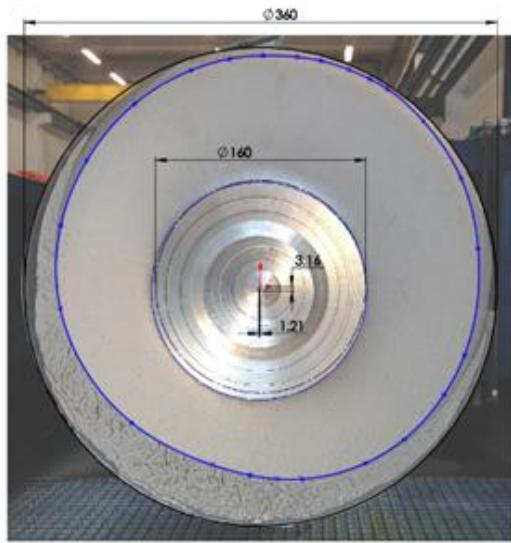


## fractured rod of the hydraulic cylinder

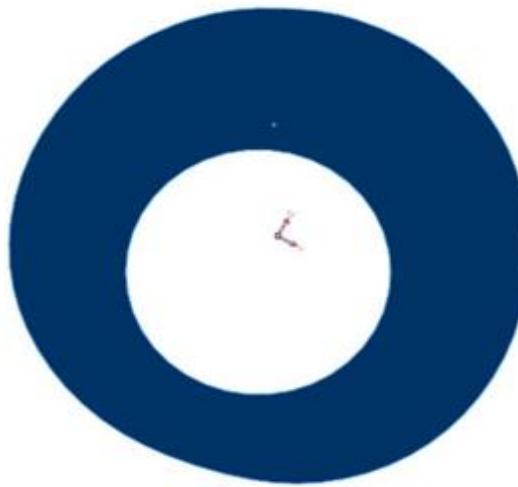


- 42CrMo4 steel ( $\sigma_{UTS}=830$  MPa;  $\sigma_{YS}=621$  MPa)
- length: 3990 mm
- external diameter: 340 mm
- internal diameter: 165 mm

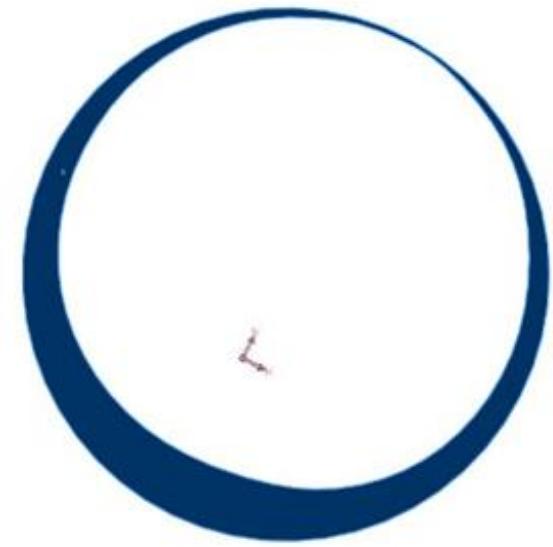
## fractured rod of the hydraulic cylinder



fatigue and final  
rupture surface



fatigue surface area



final rupture area

it was noticed that the hollow cylinder is not precisely concentric and fatigue surface area indicates that the cylinder was loaded with some bending stress (study performed using Solidworks)

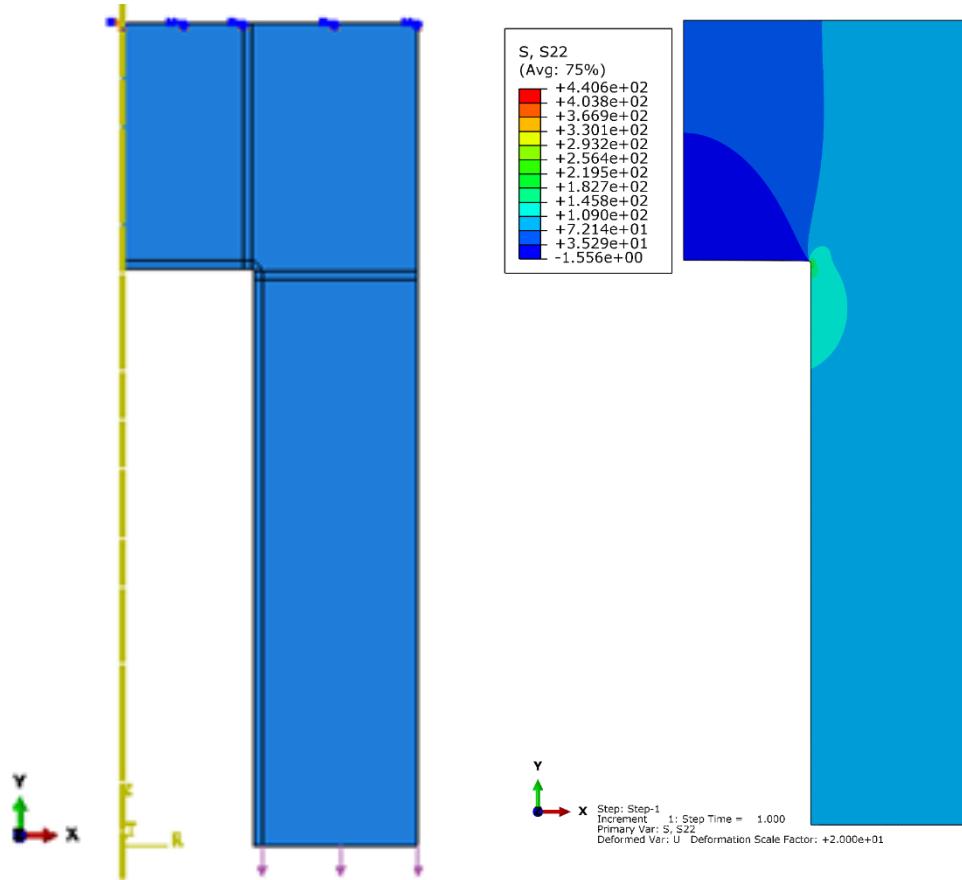
**two common approaches that have been employed to design shafts based on standards are:**

- (i) ANSI/ASME B106.1M, “Design of Transmission Shafting”, last edition in 1984**
- (ii) DIN 743, “Calculation of load capacity of shafts and axles” (German standard), last edition in 2012**

**the ASME approach is based upon a concept of static equivalent stress using Soderberg’s criterion**

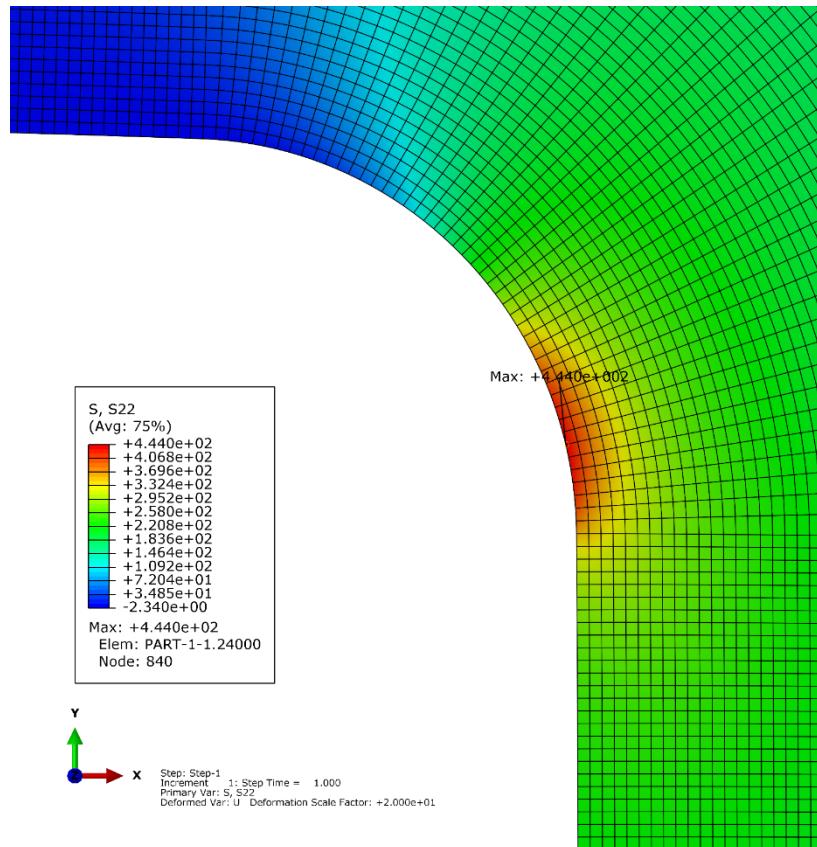
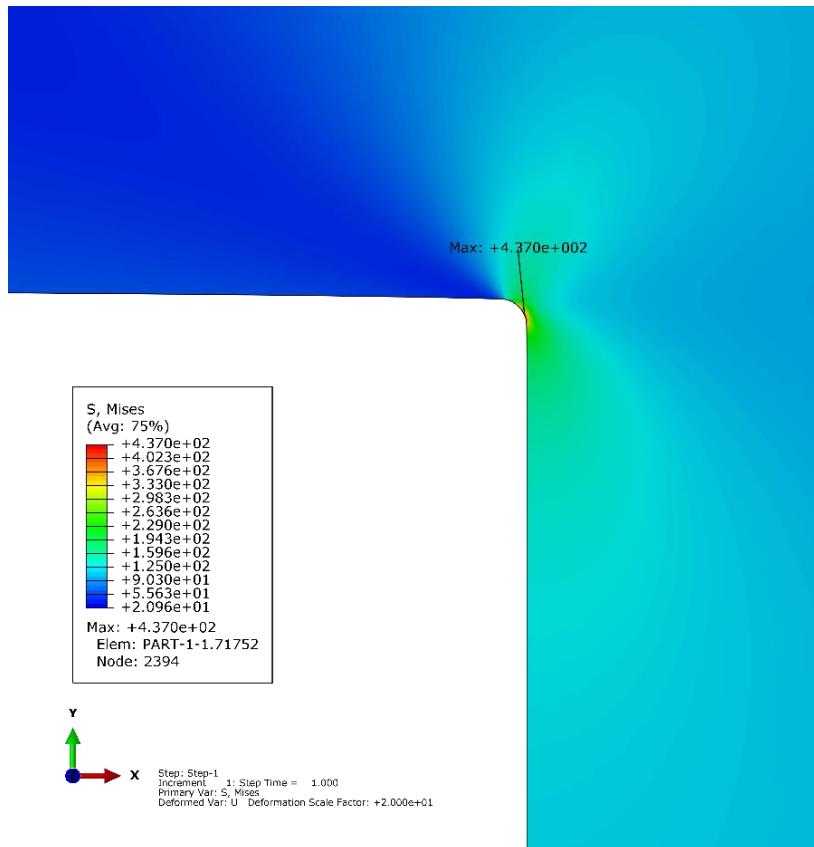
**the DIN procedure for shaft fatigue design is based upon the amplitude of the normal stress and the amplitude of the shear stress. The normal stress is separated into its components resulting from axial load and from bending**

## Stress concentration factor



- **for the stress concentration factor determination, finite elements were performed in Abaqus with axisymmetric elements**
- **in the different rod radius analyses, different element sizes were evaluated in order to study the mesh sensitivity in the stress concentration factor**

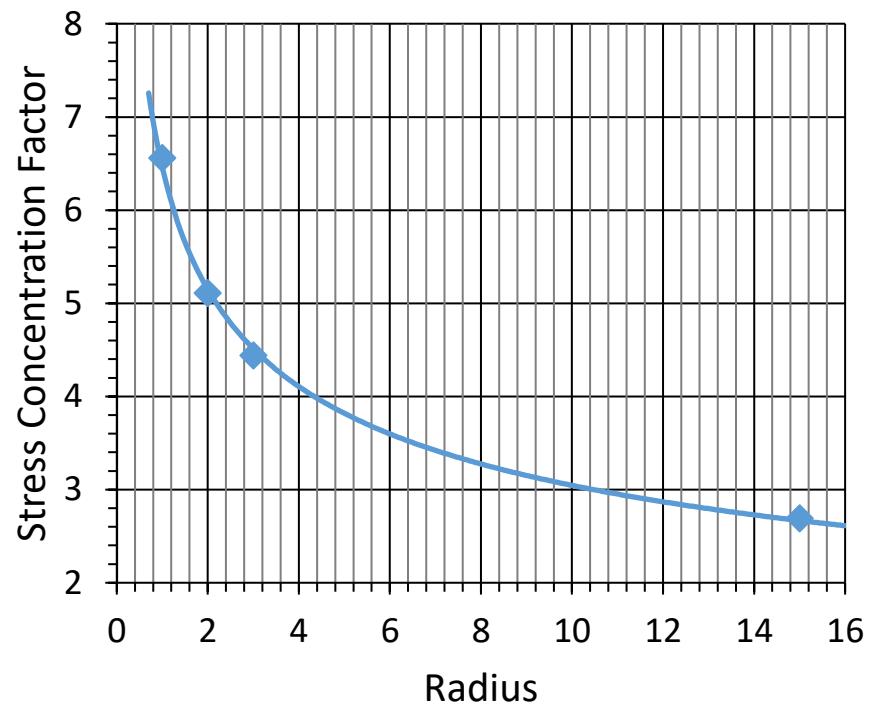
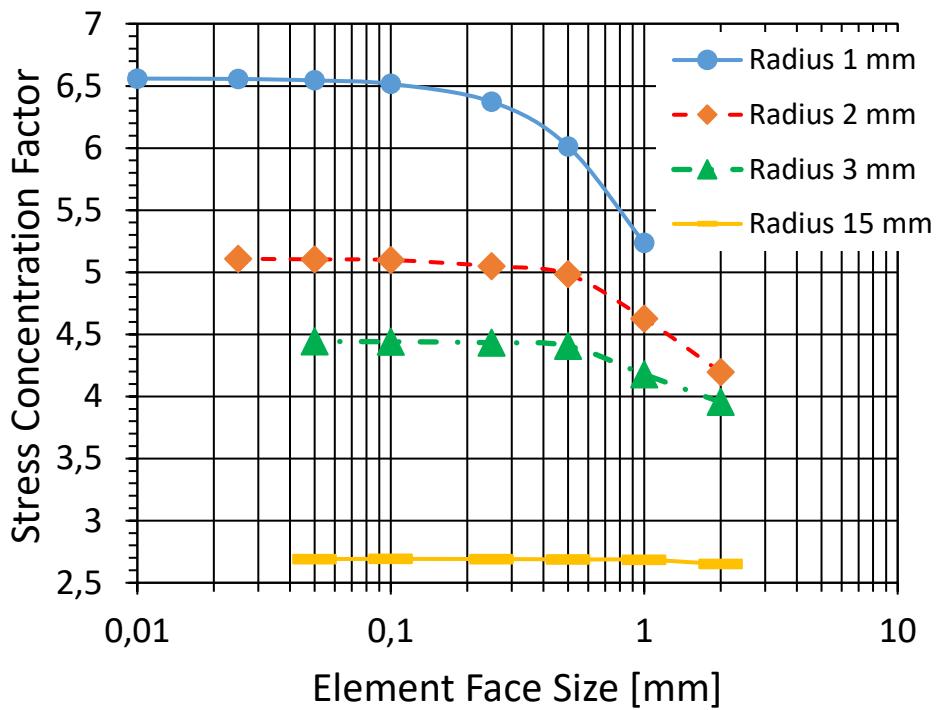
## Stress concentration factor



**von Mises stress field and  $\sigma_{yy}$  in the critical point**

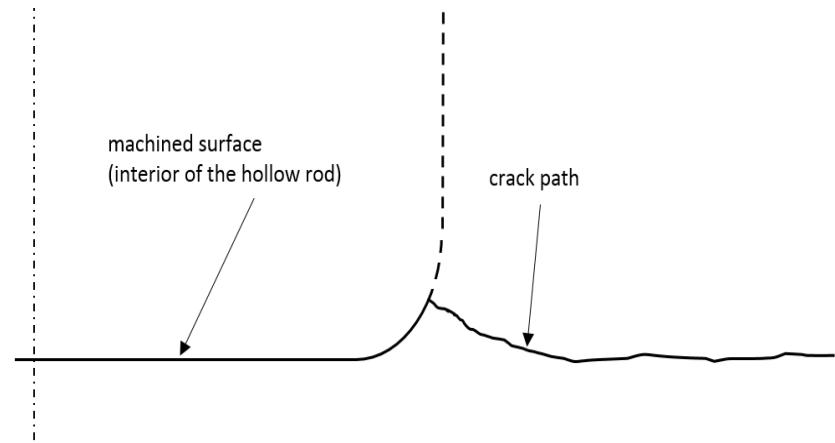
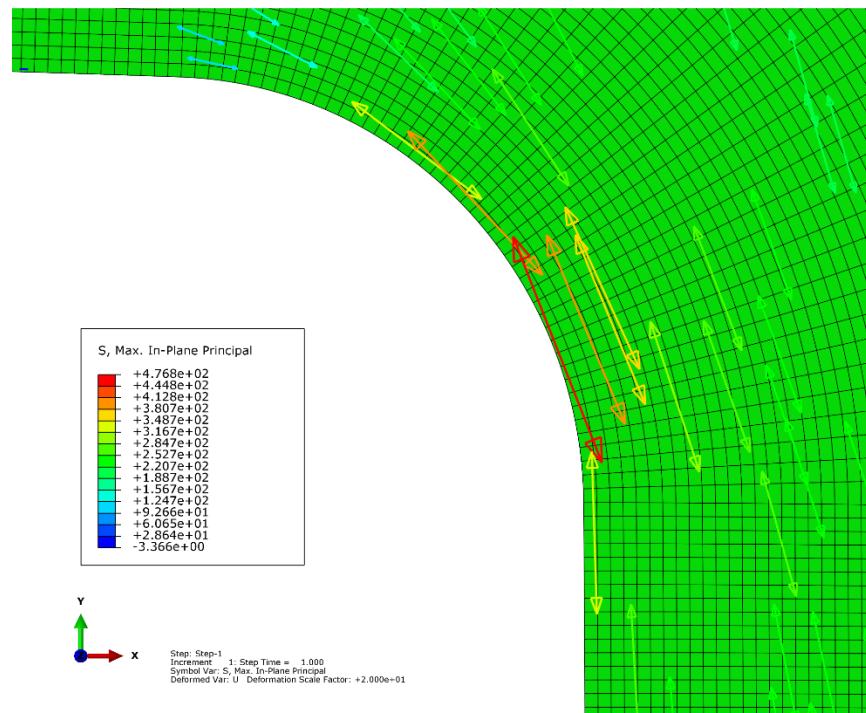
## Stress concentration factor

### stress concentration factor calibration



**the evolution of the concentration factor for the different radii shows that using quadratic elements, the element face width should be 10 times less than the radius**

## Stress concentration factor

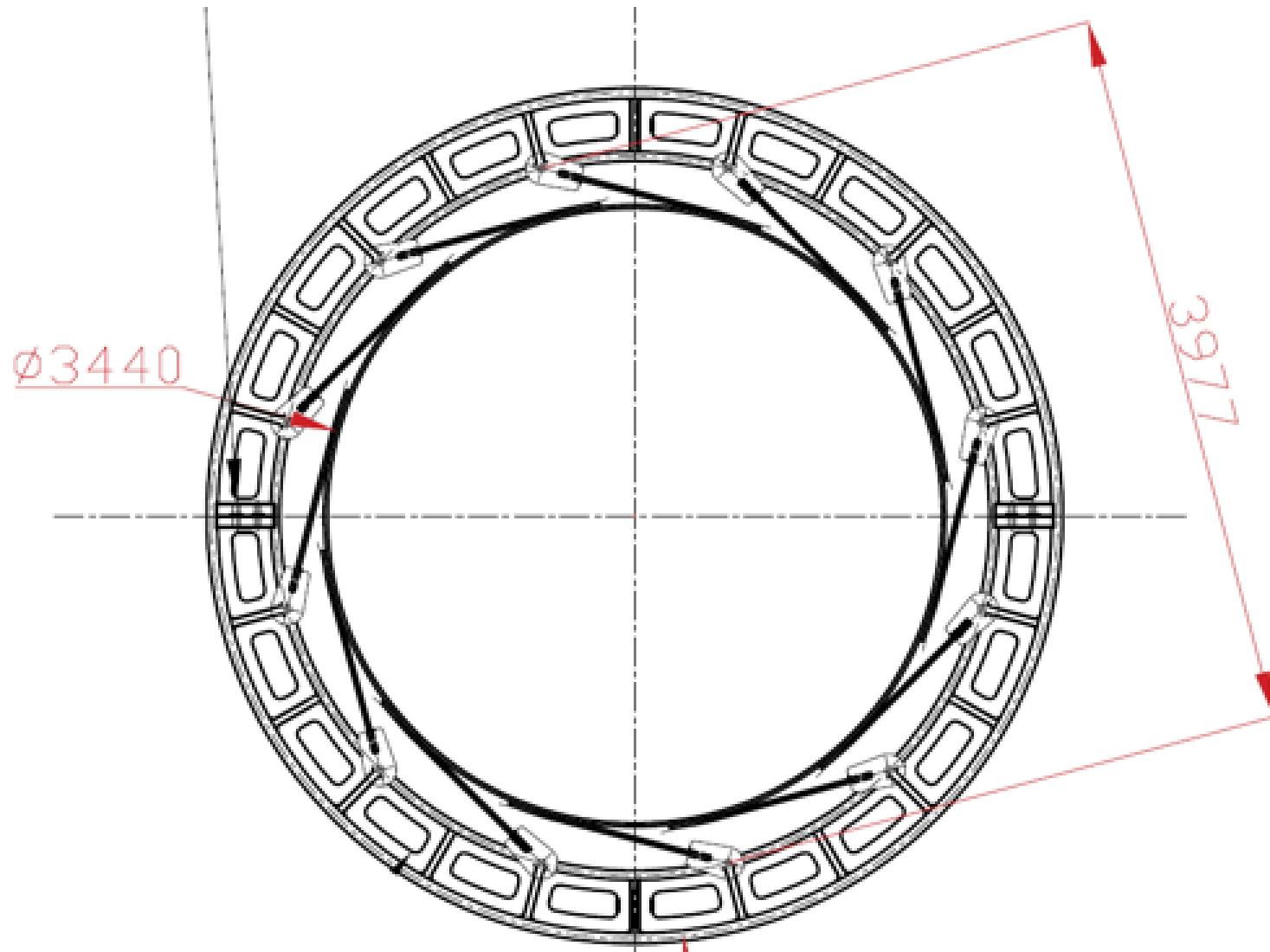


**the direction of maximum stress and the ledge on the fracture and the respective positions are in accordance**

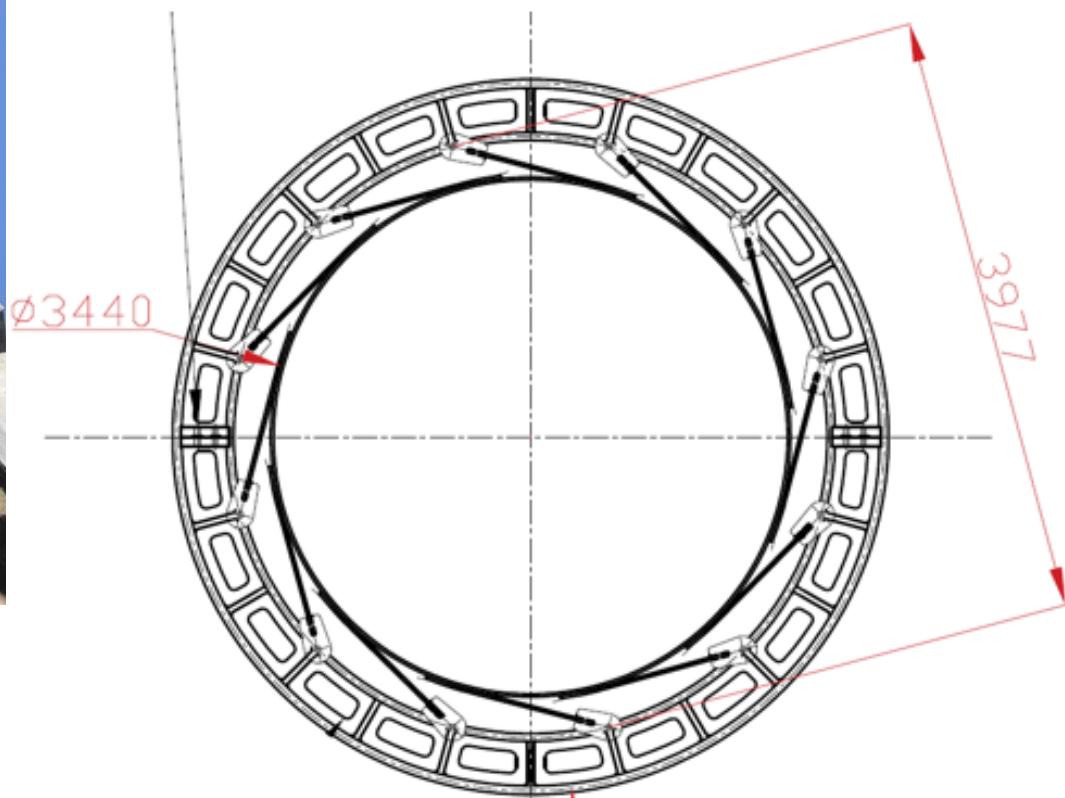




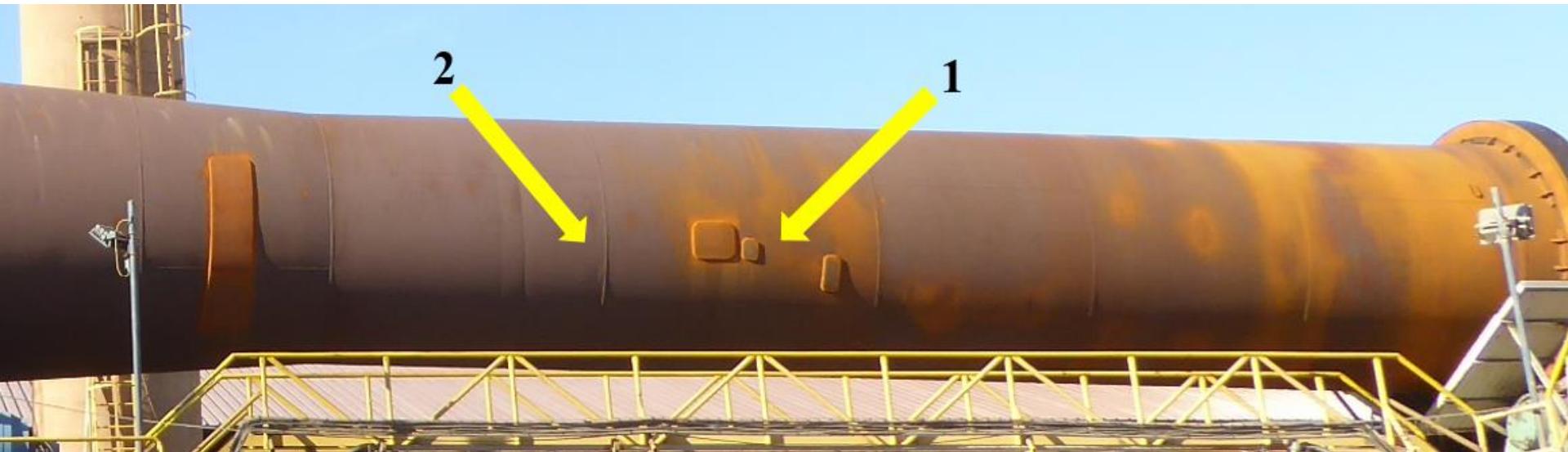
Bernardo F. de Mendonça, Paulo M.S.T. de Castro, 'Rupture of the girth gear / kiln shell connection at an expanded clay factory', *Anales de Mecánica de la Fractura*, 2020



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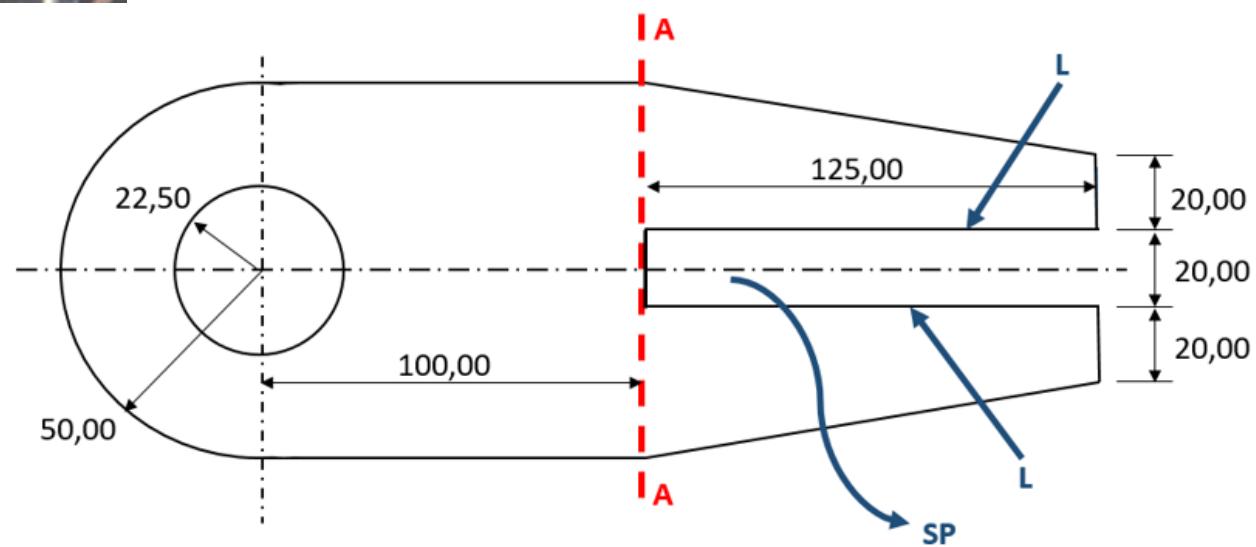
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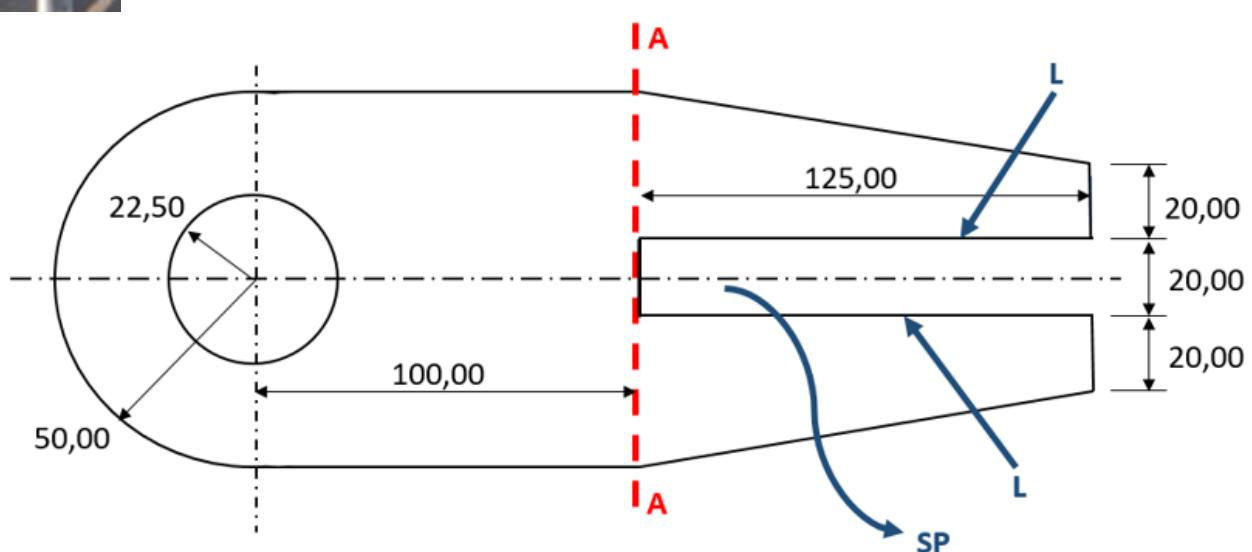
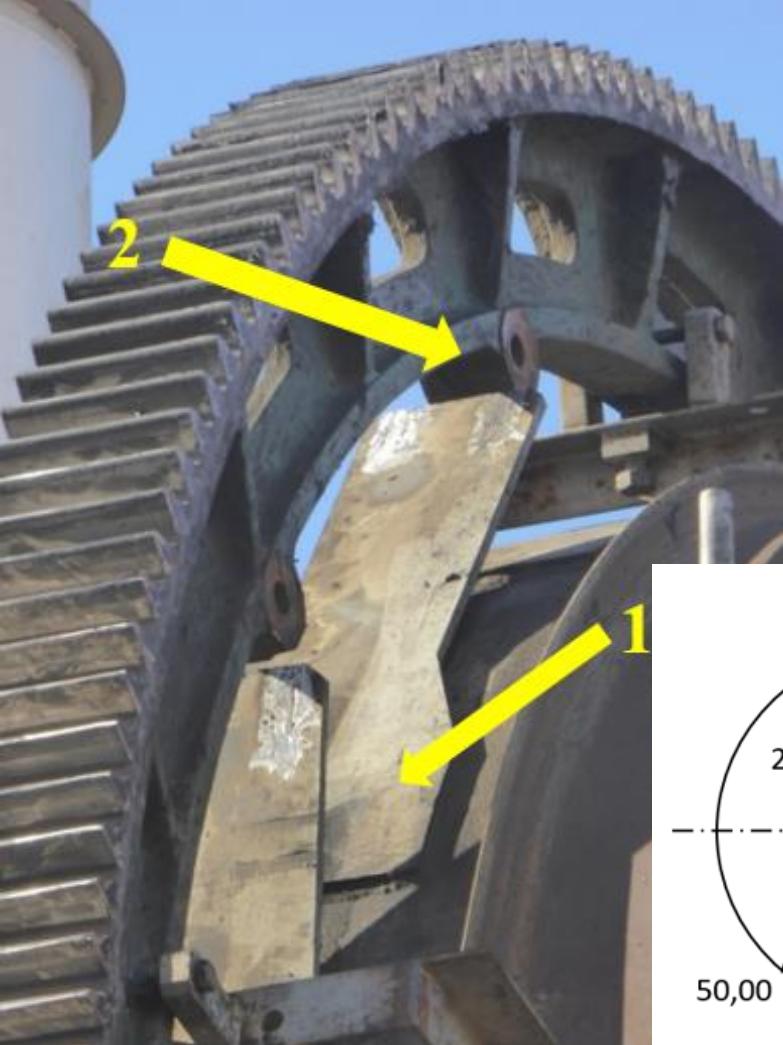
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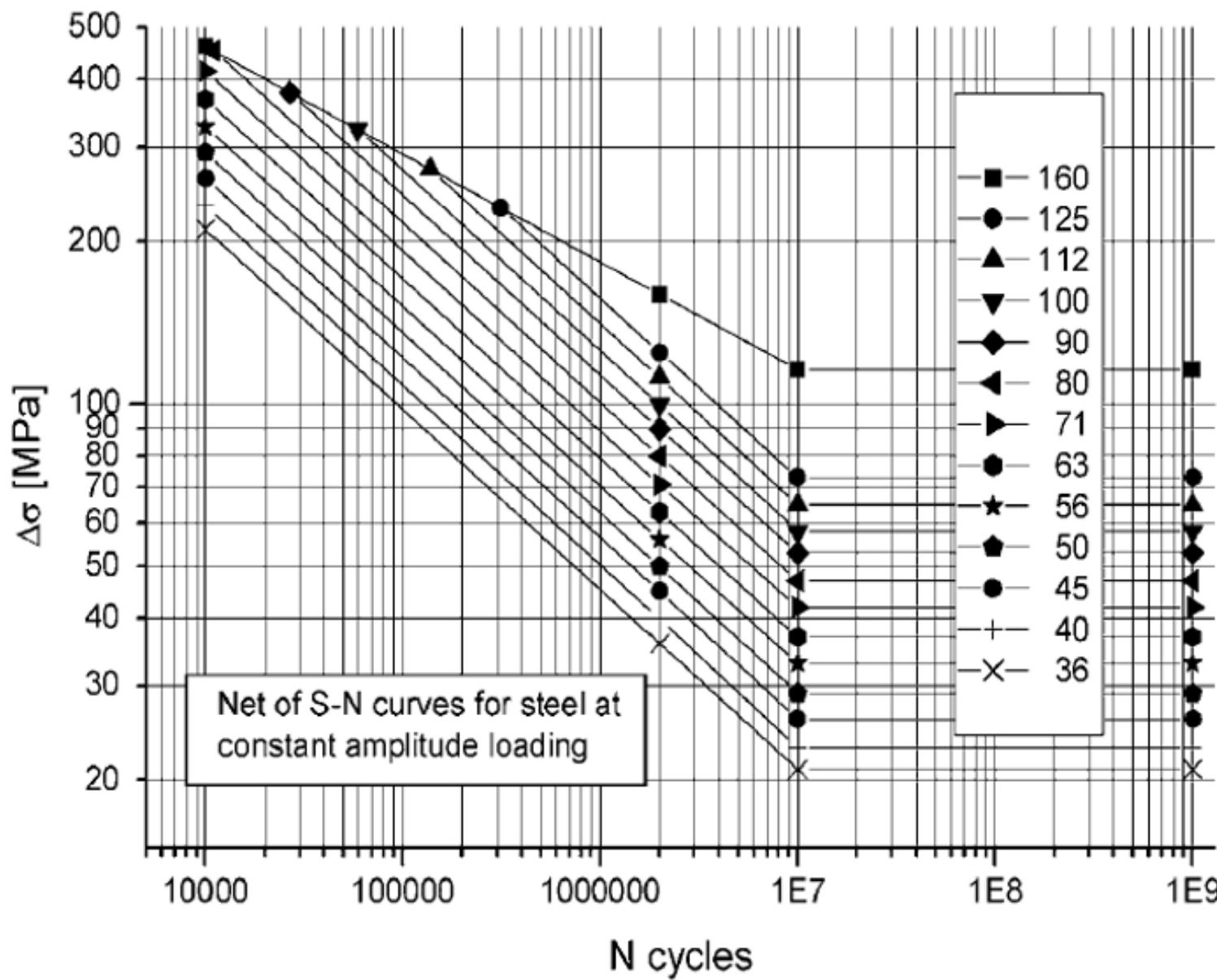
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## **estudos a várias escalas**

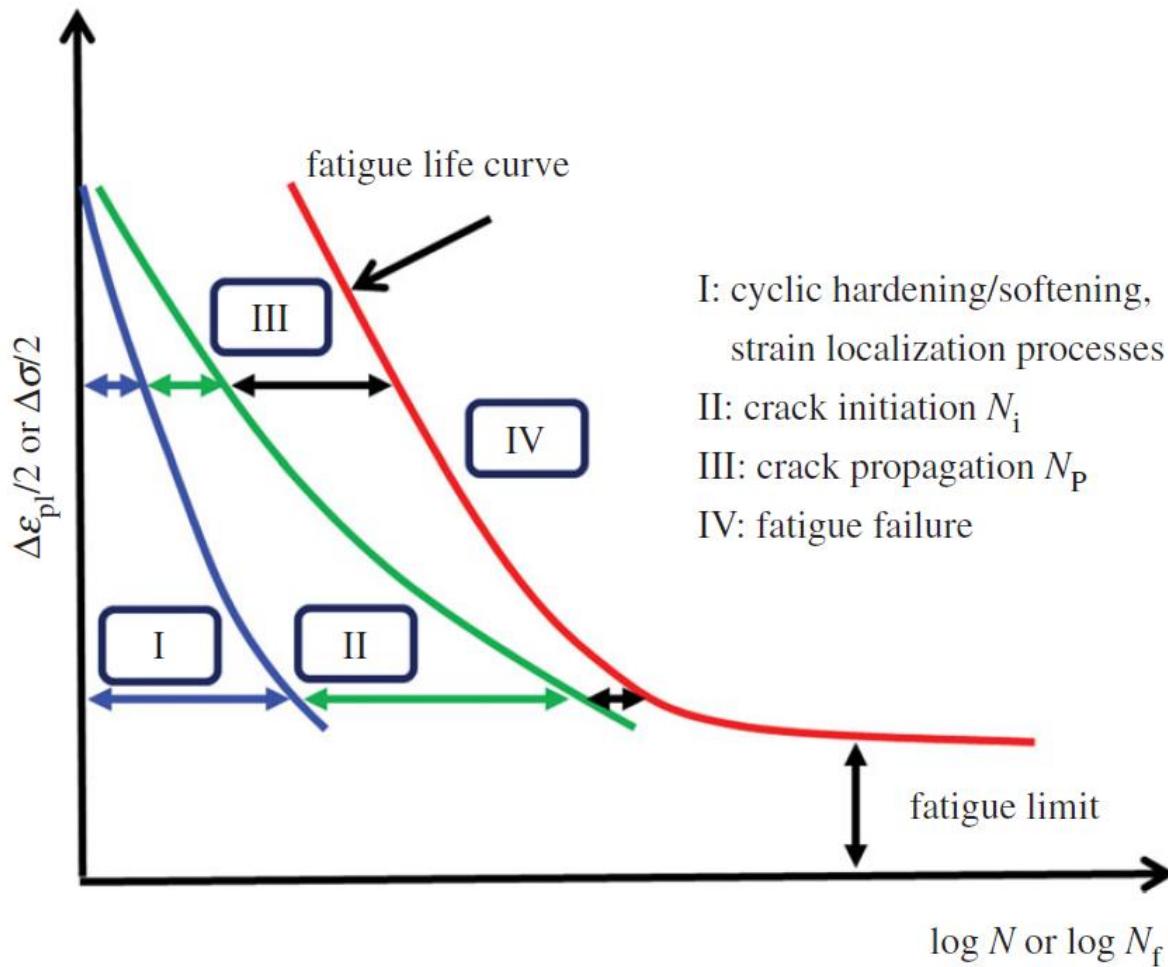
- provete
- ligação estrutural
- módulo estrutural
- estrutura completa

.....

## **e com vários objectivos**

- iniciação de fendas
- propagação de fendas
- resistência residual

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H. Mughrabi, 'Microstructural mechanisms of cyclic deformation, fatigue crack initiation and early crack growth', *Philosophical Transactions of the Royal Society A*, vol.373, (2038), 2015

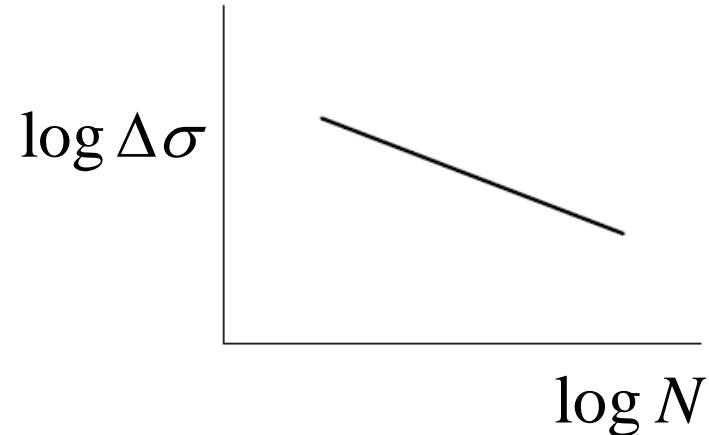
## problemas

iniciação

- Wöhler, curvas SN baseadas em tensões elásticas

propagação

- lei de Paris



## entre outros desenvolvimentos

iniciação

- estudos elasto-plásticos: Coffin-Manson, Neuber, ....

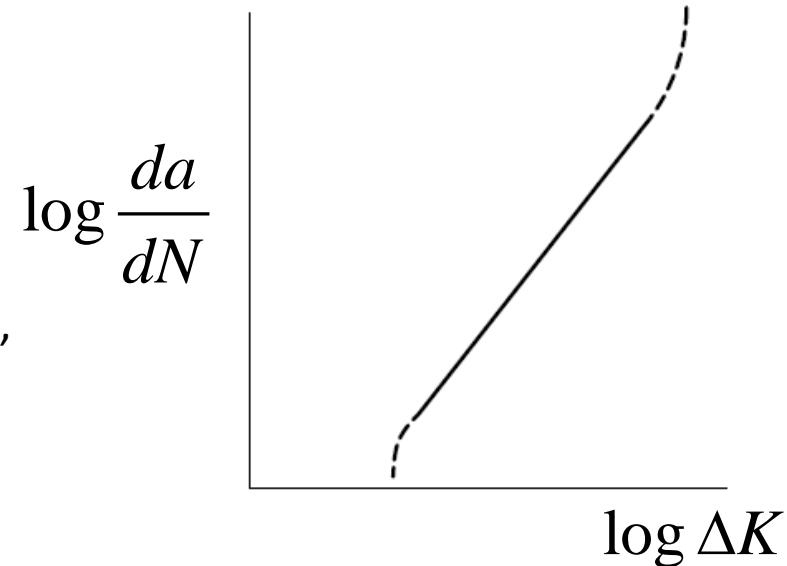
propagação

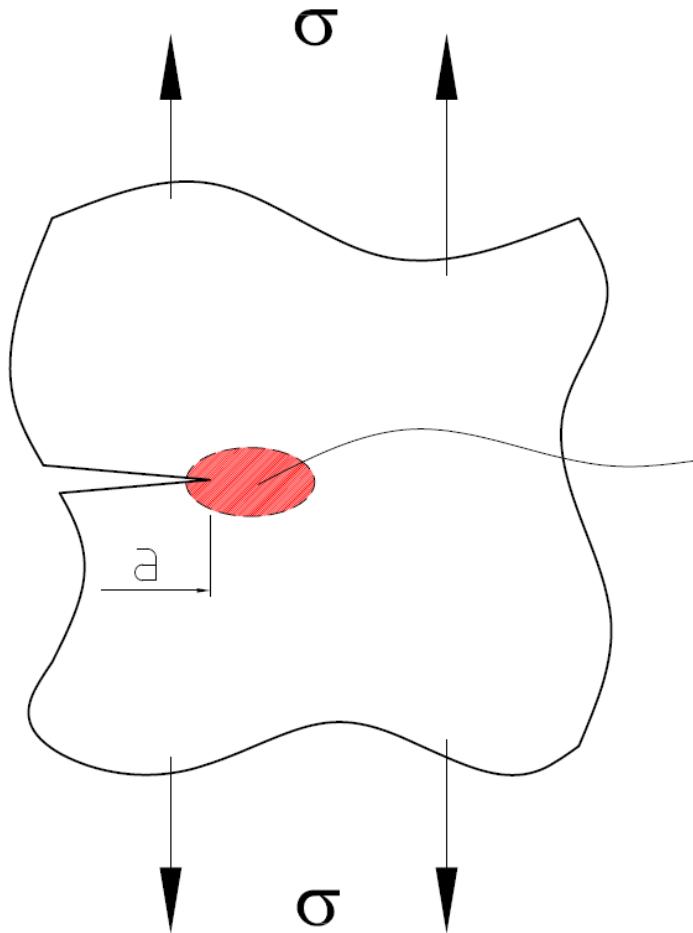
- razão de carga  $R = \text{carga max load}/\text{carga min}$ , limiare de propagação, ....

Acumulação de dano

- Miner, ....

.....





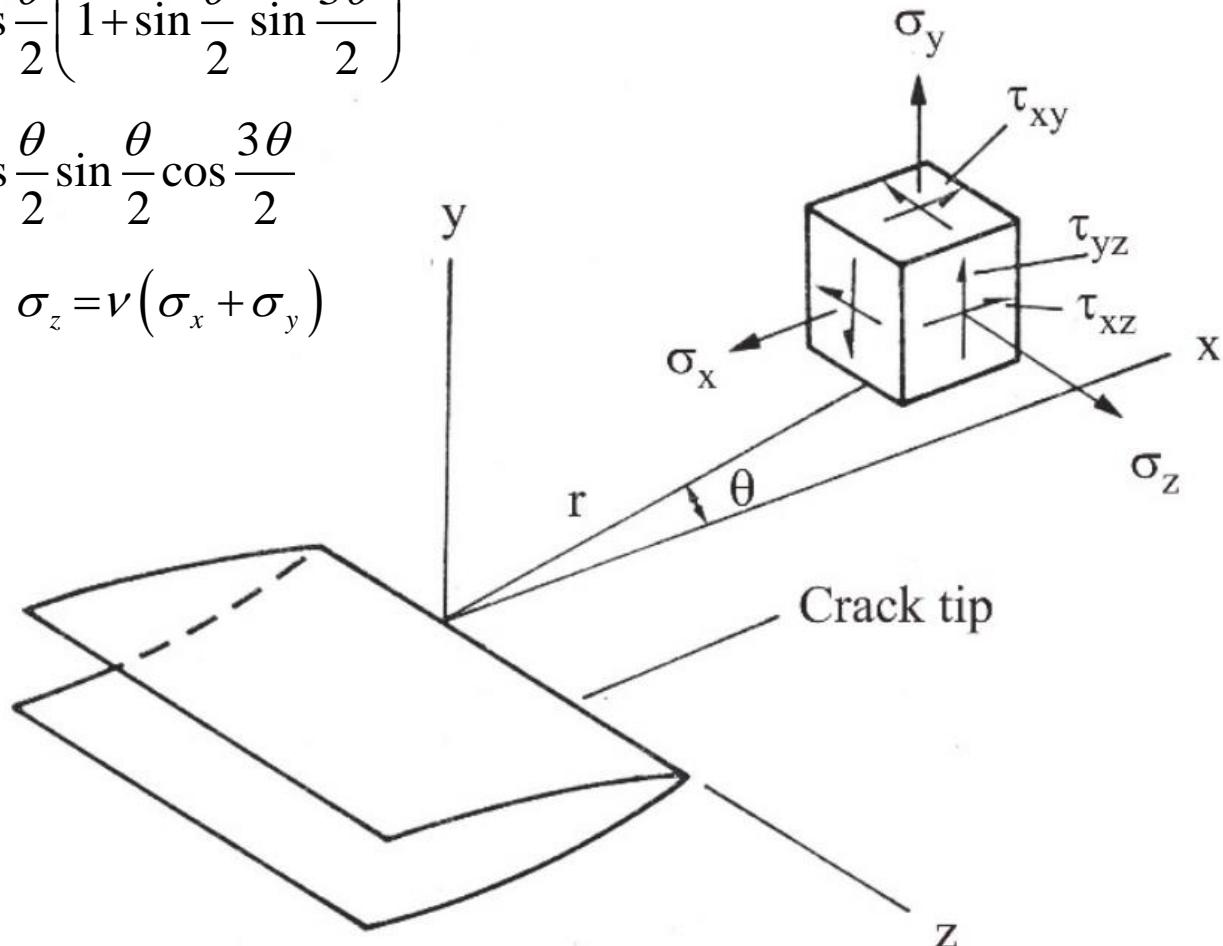
$$\sigma_{local} = f(K)$$
$$K = Y\sigma\sqrt{\pi a}$$

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cos \frac{3\theta}{2}$$

$$\sigma_z = 0 \quad , \quad \text{ou} \quad \sigma_z = \nu (\sigma_x + \sigma_y)$$

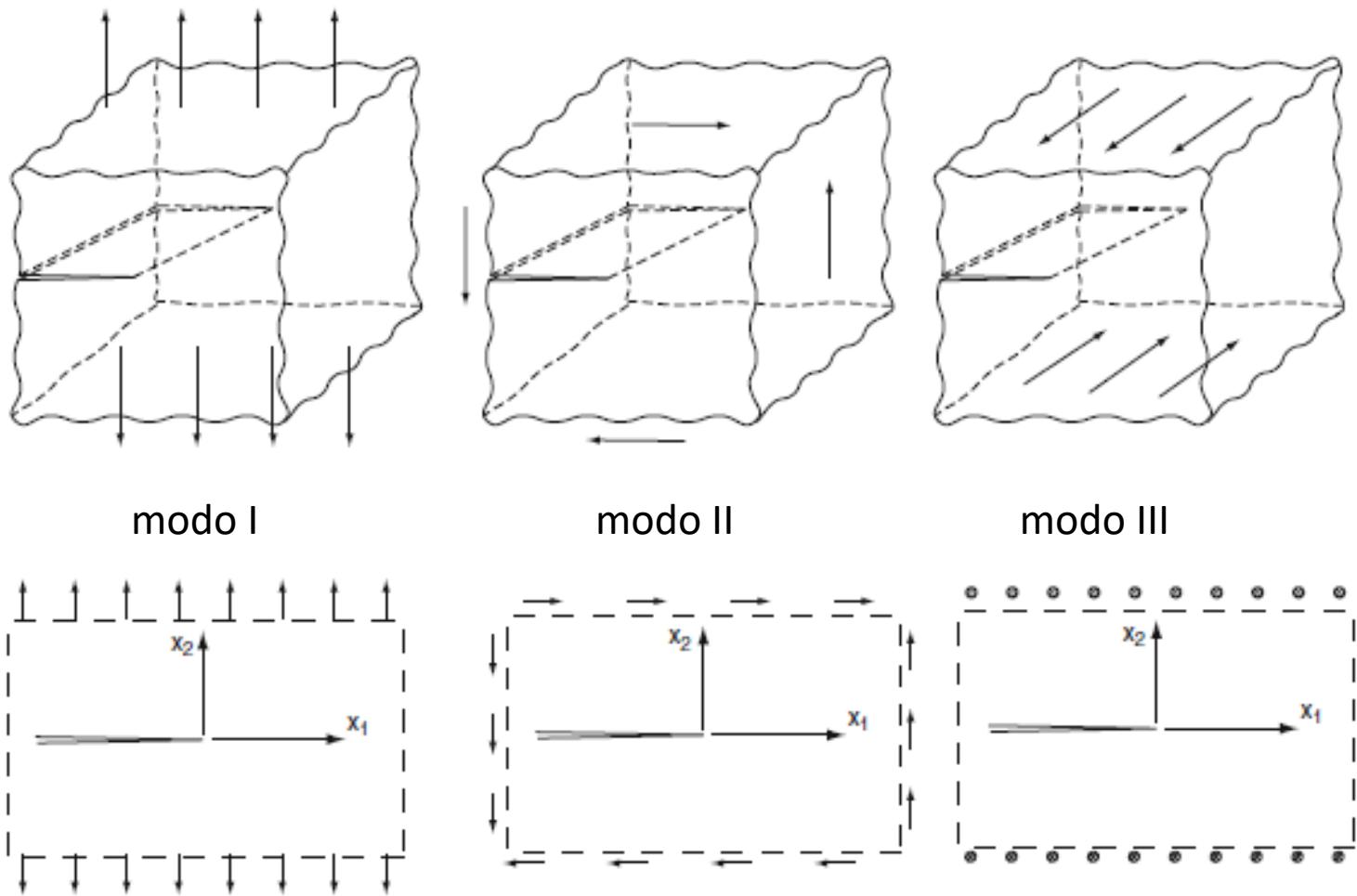


**carregamentos:  
forças, deslocamentos, condições fronteira**

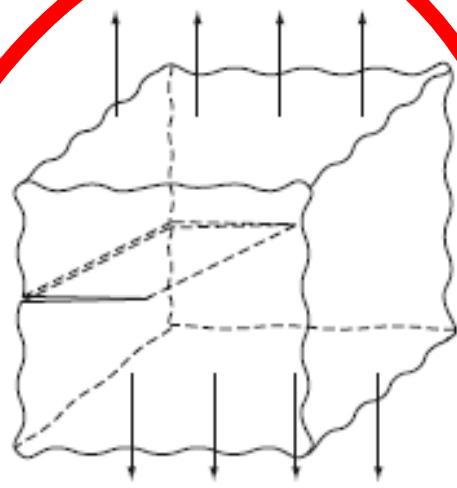


**fendas  
comp. 'a', ou geometria...**

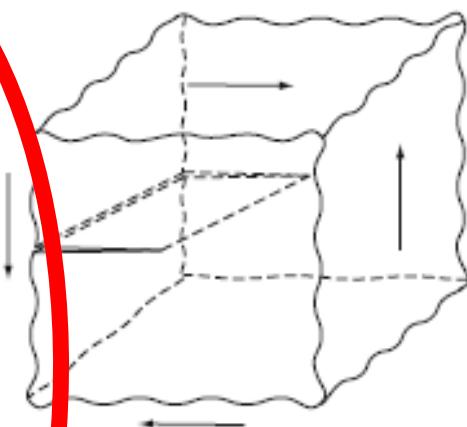
**propriedades mecânicas  
tensão de ced., tenacidade,  
...**



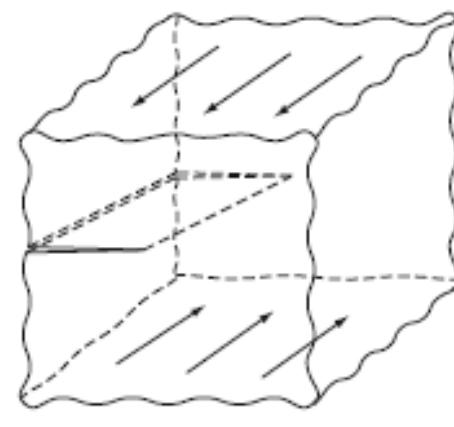
A.T. Zehnder, 'Modes of fracture', in: Q.J. Wang, Y.-W. Chung, eds., 'Encyclopedia of Tribology', Springer, pp.2292-2295, 2013



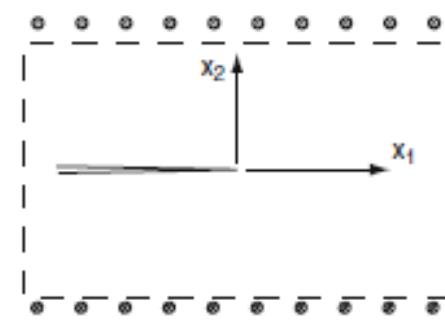
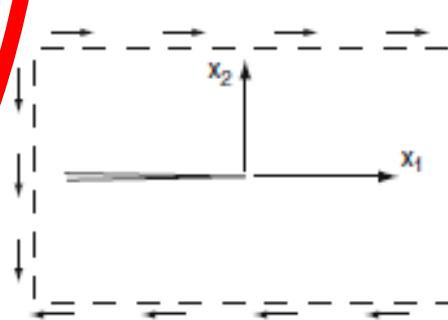
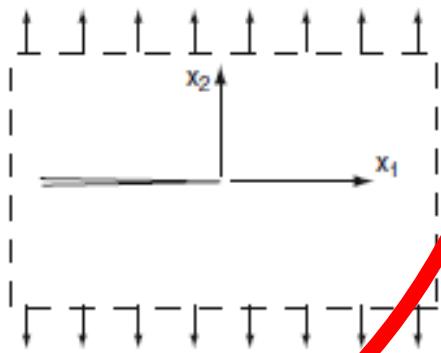
Mode-I (tension)



Mode-II (in-plane shear)



Mode-III (anti-plane shear)



A.T. Zehnder, 'Modes of fracture', in: Q.J. Wang, Y.-W. Chung, eds., 'Encyclopedia of Tribology', Springer, pp.2292-2295, 2013

$$K = f(\sigma, a, \dots) = Y\sigma\sqrt{\pi a}$$

$$\Delta K = f(\Delta\sigma, a, \dots) = Y\Delta\sigma\sqrt{\pi a}$$

$$\frac{da}{dN} = f(\Delta K, \dots)$$

$$N = \int_{a_i}^{a_f} \frac{da}{f(\Delta K)}$$

sendo aplicável a lei de Paris:  $\frac{da}{dN} = C(\Delta K)^m$ , e com  $Y \approx const$

$$\begin{aligned} N &= \int_{a_i}^{a_f} \frac{da}{C(\Delta K)^m} = \int_{a_i}^{a_f} \frac{da}{C(\Delta K)^m} = \frac{1}{C(Y\Delta\sigma\sqrt{\pi})^m} \int_{a_i}^{a_f} \frac{da}{a^{m/2}} = \\ &= \frac{2 \left( a_f^{\frac{2-m}{2}} - a_i^{\frac{2-m}{2}} \right)}{(2-m)C(Y\Delta\sigma\sqrt{\pi})^m} \end{aligned}$$



C. Moura Branco • A. Augusto Fernandes • Paulo M. S. Tavares de Castro

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# Damage Tolerance of Metallic Aircraft Structures

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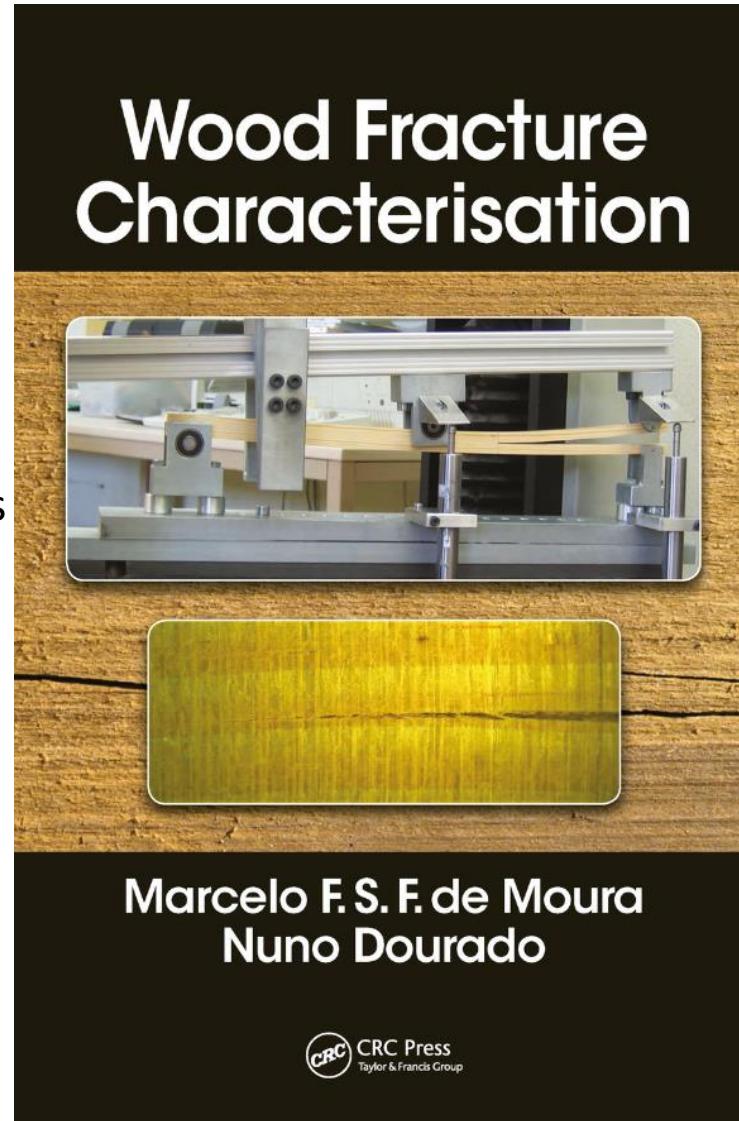
**Mecânica da Fratura e Fadiga** Exemplos de cálculo e aplicação

2014

colegas da FEUP  
trabalhando  
nestes temas  
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entre outros,

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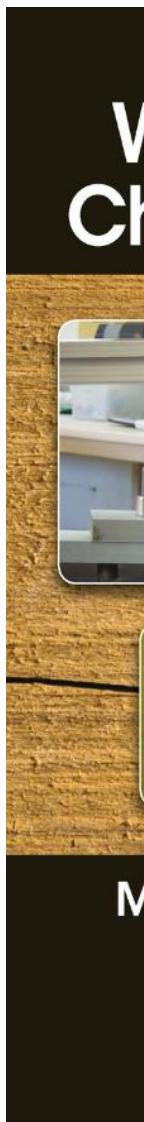
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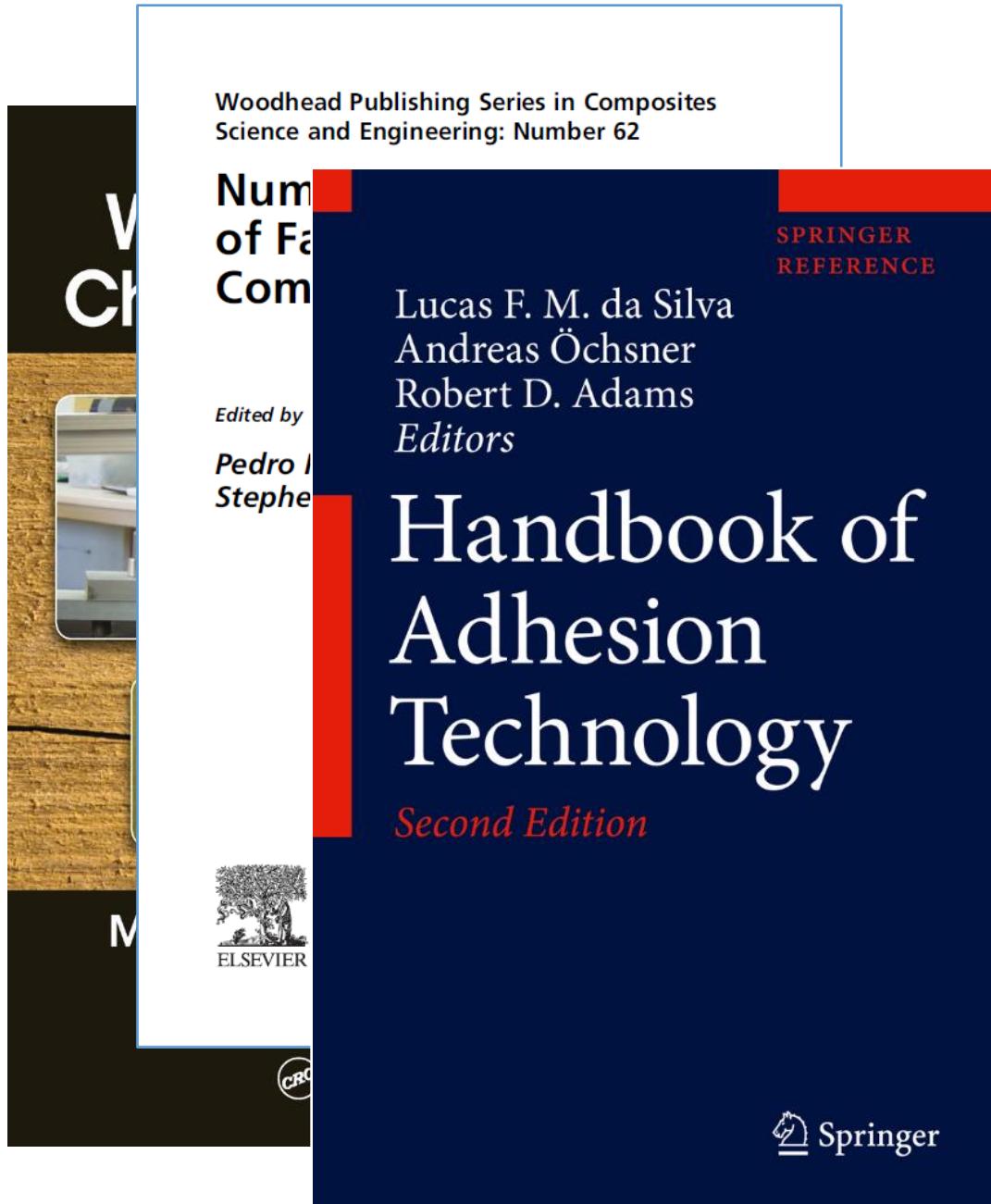


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## National Transportation Safety Board

Washington, D.C. 20594

Accident Number: DCA11MA039  
Operator/Flight Number: Southwest Airlines, Flight 812  
Aircraft and Registration: Boeing 737-3H4, N632SW  
Location: Yuma, Arizona  
Date: April 1, 2011  
Adopted: September 24, 2013

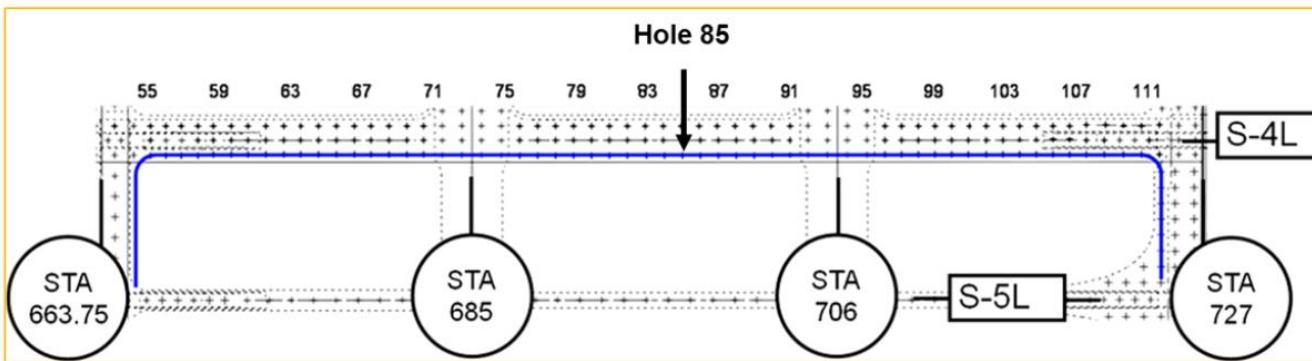
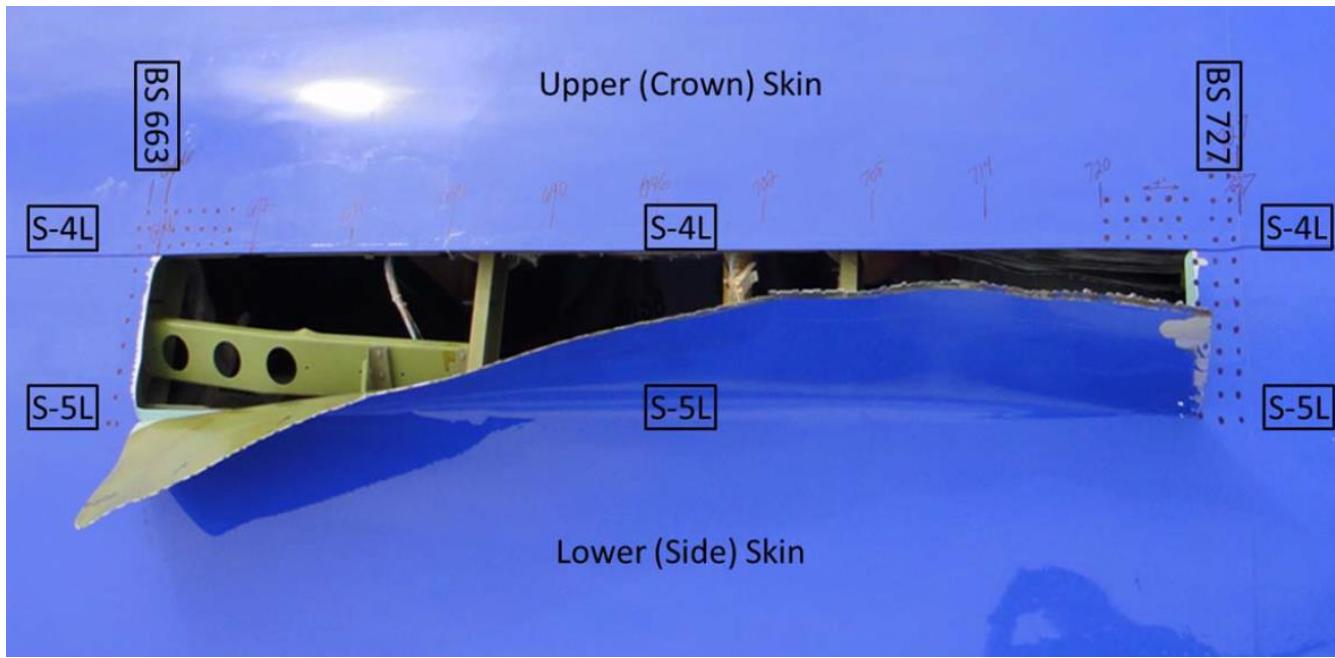
### HISTORY OF FLIGHT

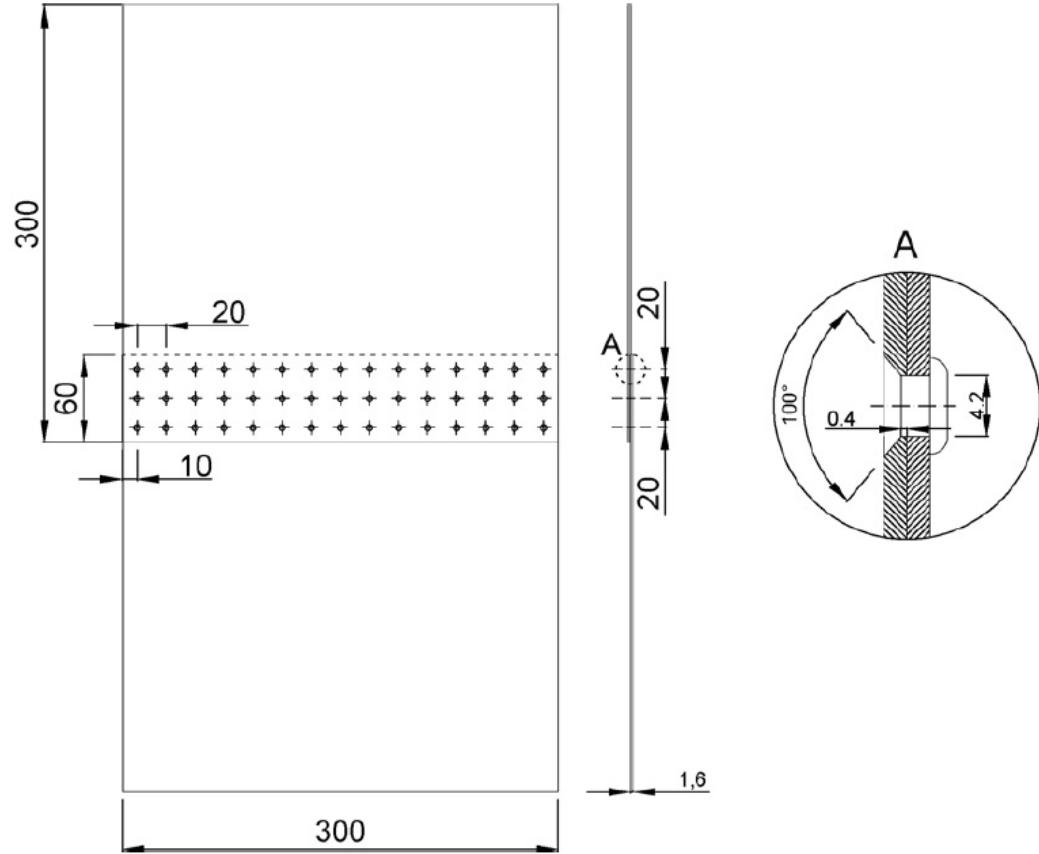
On April 1, 2011, about 1558 mountain time, Southwest Airlines flight N632SW, operating as Southwest Airlines flight 812, was climbing through flight level 340 en route to Yuma International Airport (NYL), Yuma, Arizona. There were two passengers on board, one crewmember and one nonrevenue passenger. One passenger received minor injuries. The airplane sustained substantial damage to the upper left side of the fuselage, specifically a section of fuselage skin about 60 inches long by 12 inches wide, located on the upper left side above the wing. The flight departed Phoenix Sky Harbor International Airport, Phoenix, Arizona, at 1558, en route to Sacramento, California.

According to the flight crew and records, at 1558:05, an unidentified sound was recorded in the cabin. At 1558:07, 2 seconds later, the captain announced that the airplane was descending for oxygen masks on; sounds consistent with decompression were heard on the voice recording. The captain declared an emergency descent to 11,000 feet. The air traffic controller provided vectors to descend the airplane to 11,000 feet within 5 minutes. At 1605, the cabin crew began relaying condition reports from the cabin. One passenger received a broken nose and another passenger received minor injuries. The captain declared an emergency descent to 9,000 feet, and the captain initiated the emergency descent. The airplane landed about 1620 at Yuma International Airport (NYL). The airplane landed about 1620. The passengers deplaned via airstairs.

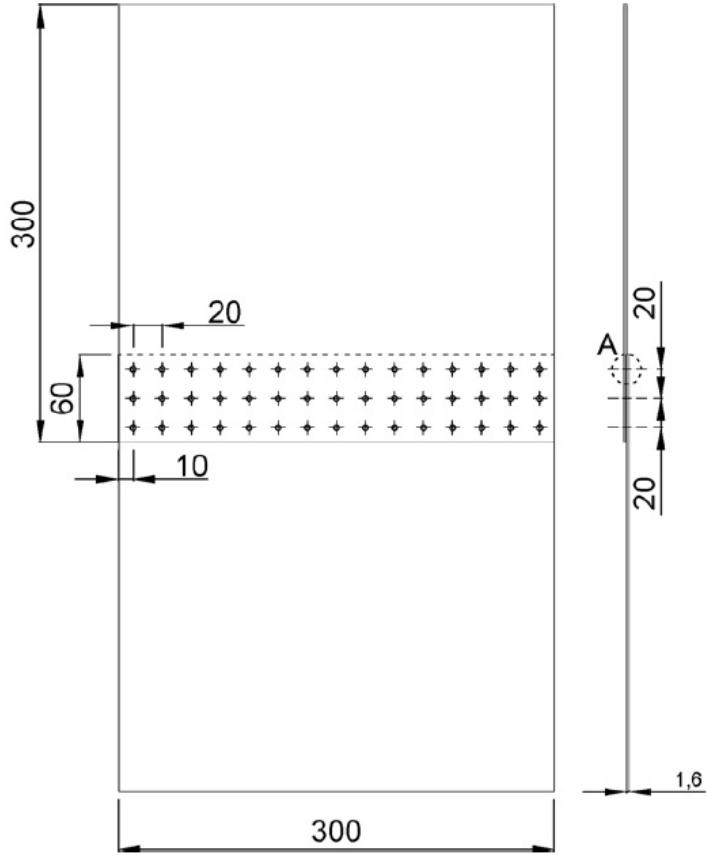


<sup>1</sup> Unless otherwise noted, all times in this brief are based on the pilot's report.

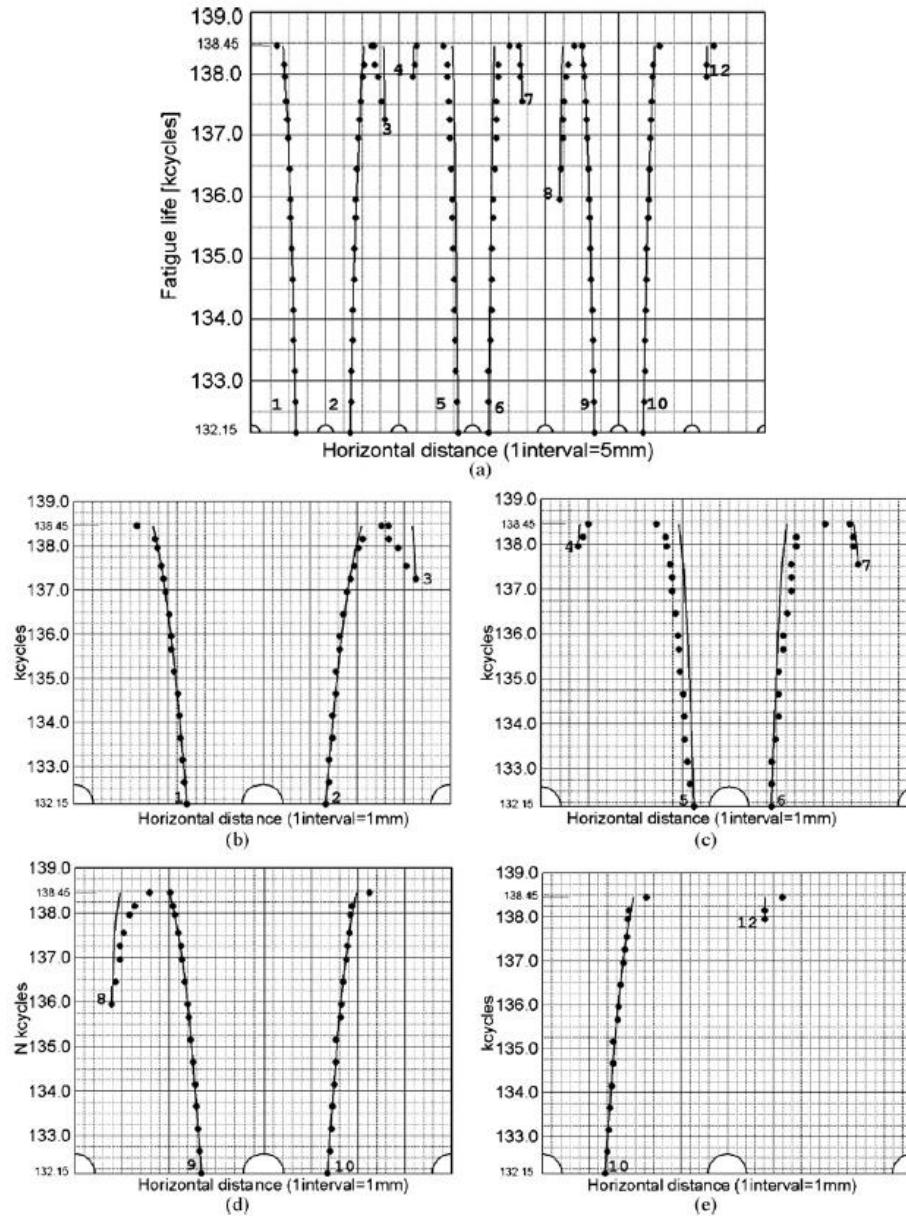


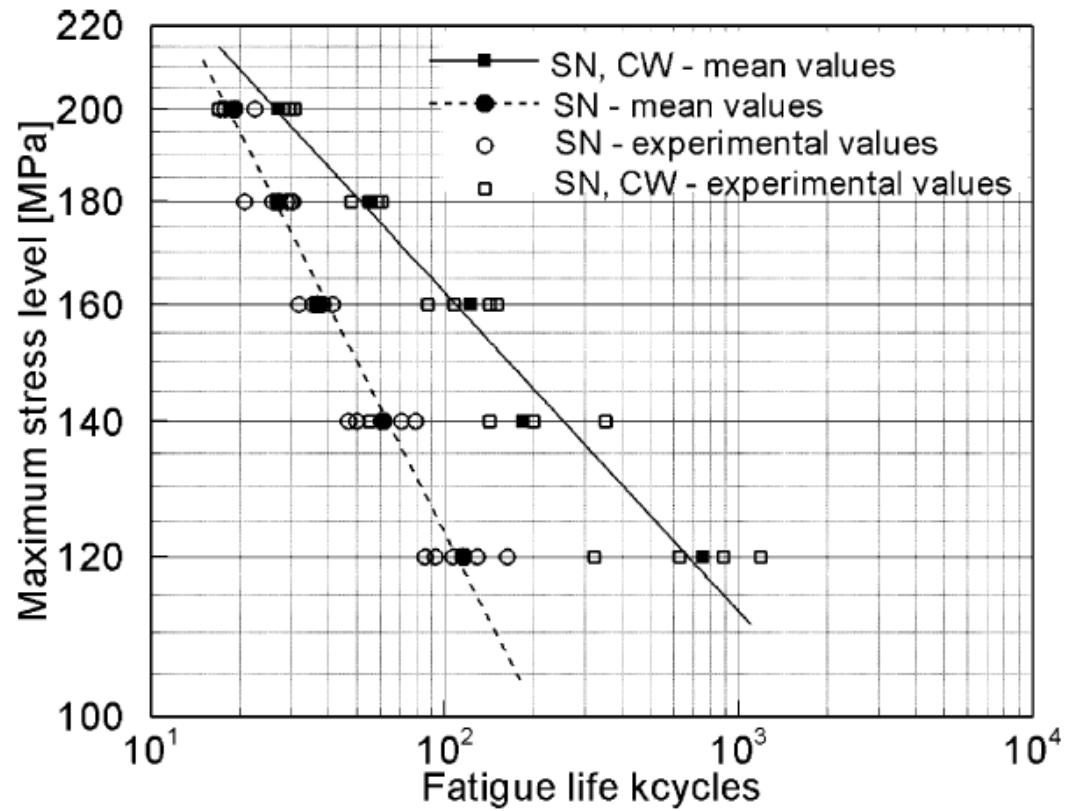
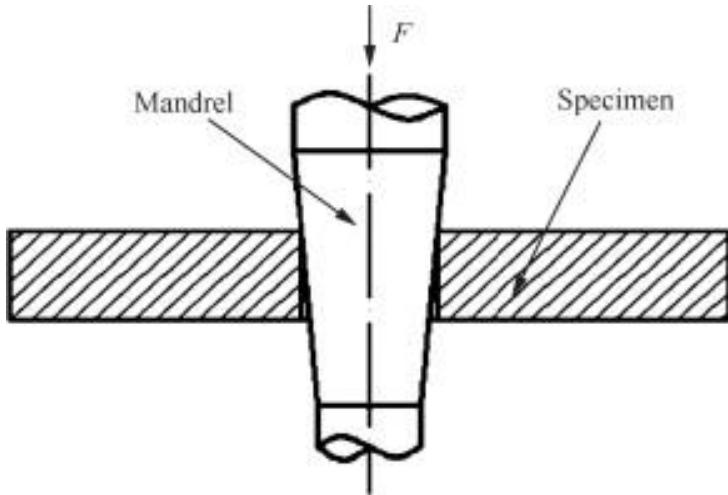


P.M.S.T. de Castro *et al.*, 'An overview on fatigue analysis of aeronautical structural details: Open hole, single rivet lap-joint, and lap-joint panel', *Materials Science and Engineering A*, vol.468–470, pp.144–157, 2007



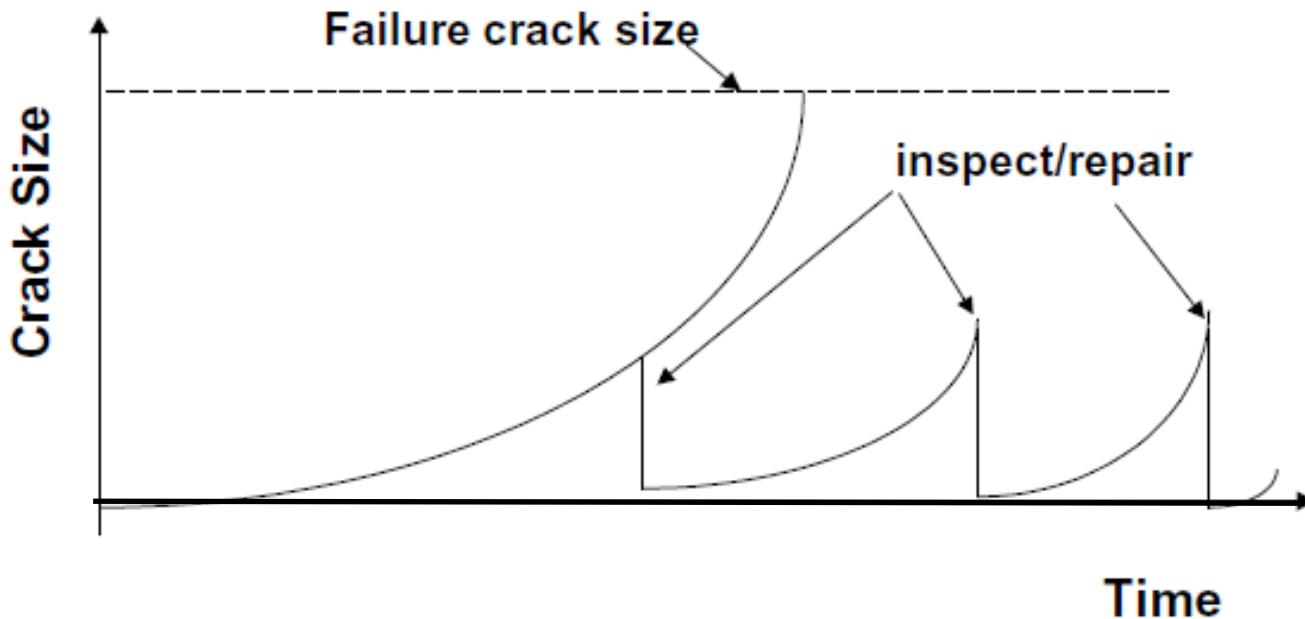
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Fu Yucan *et al.*, 'Cold expansion technology of connection holes in aircraft structures: A review and prospect', *Chinese Journal of Aeronautics*, vol.28, (4), pp.961–973, 2015

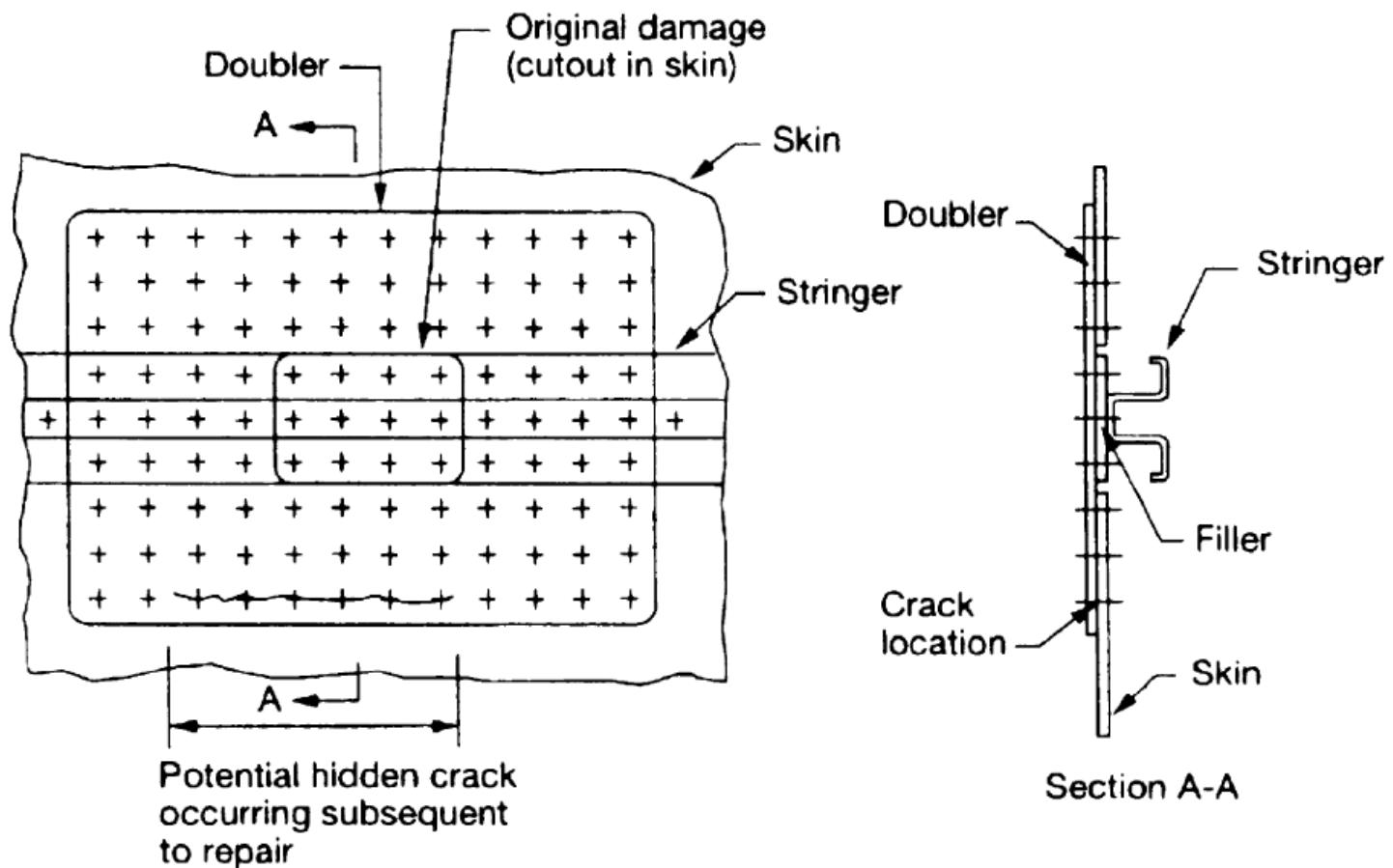
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the damage tolerance philosophy as used in aeronautical engineering  
tolerância ao dano, como usada em aeronáutica



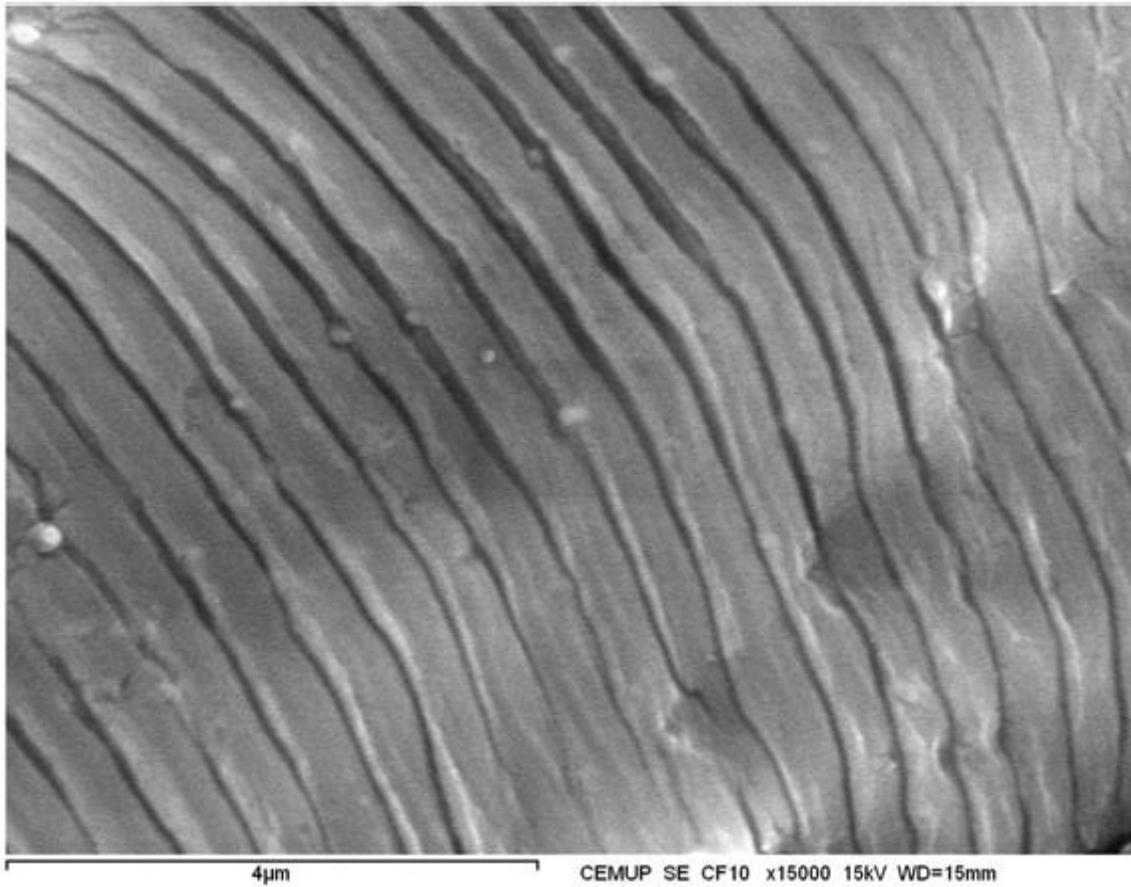
M. Chajes *et al.*, 'Steel Girder Fracture on Delaware's I-95 Bridge over the Brandywine River', Structures Congress 2005, April 20-24, 2005, NY, USA, American Society of Civil Engineers



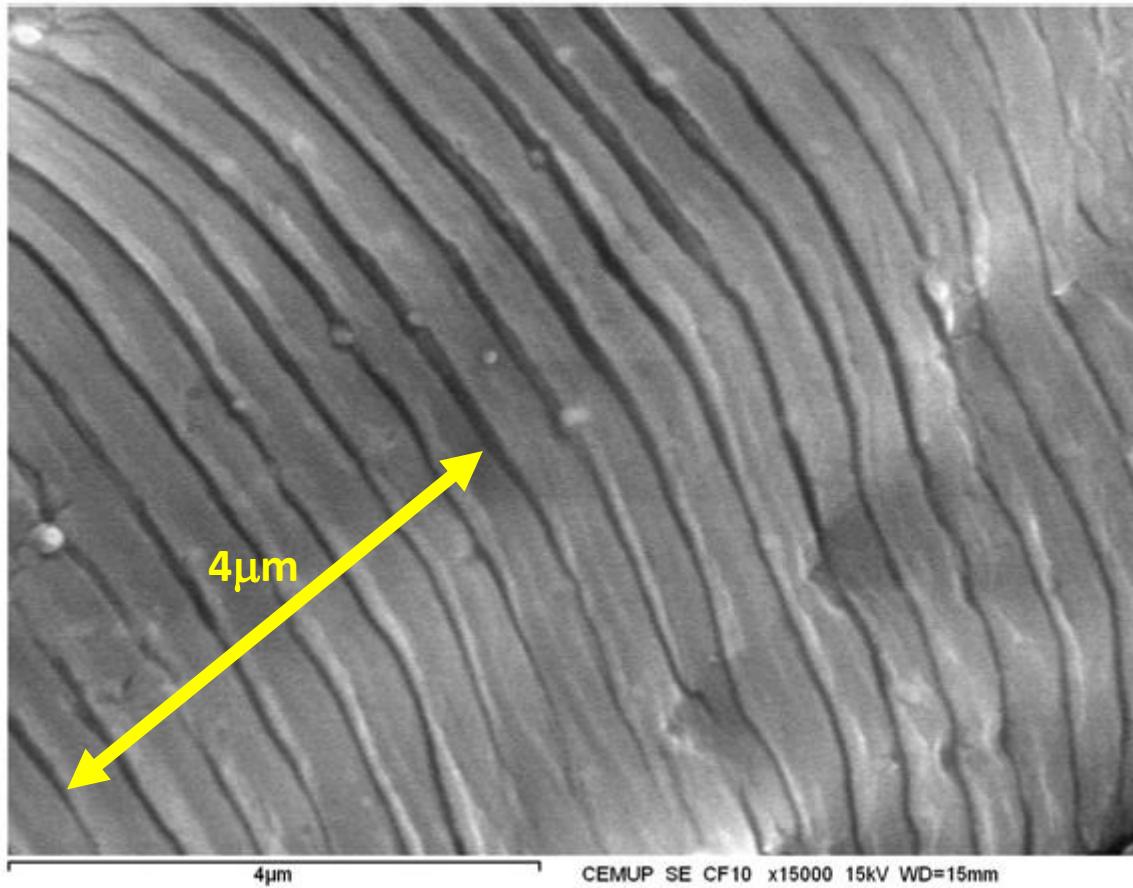
**Figure 48** Typical fuselage external skin repair

U.G. Goranson, 'Fatigue issues in aircraft maintenance and repairs', *International Journal of Fatigue*, vol.19, supp. no.1, pp.S3–S21, 1997

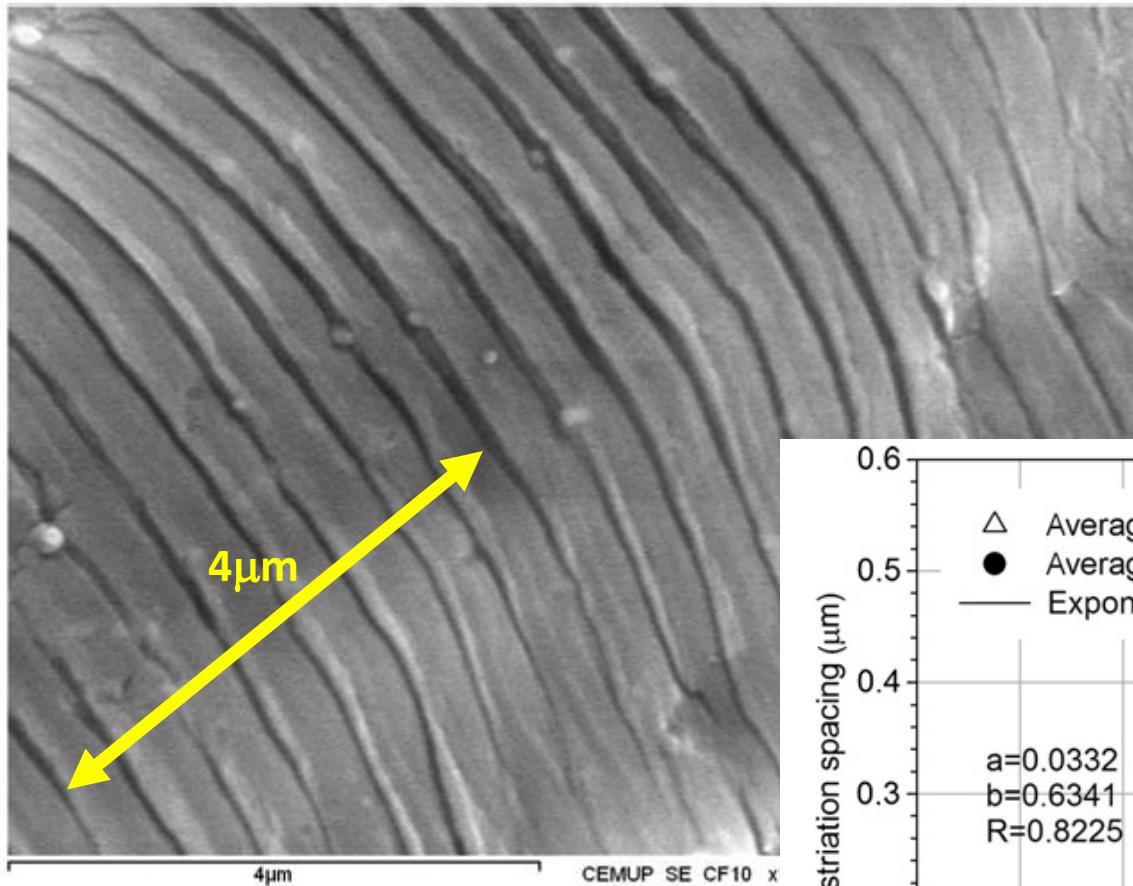




P. Moreira, M. Figueiredo, P.M.S.T. de Castro, 'Fatigue behaviour of FSW and MIG weldments for two aluminium alloys', *Theoretical and Applied Fracture Mechanics*, vol.48, pp.169–177, 2007



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estria:  $4 \mu\text{m} / 8 \approx 0.5 \mu\text{m}$

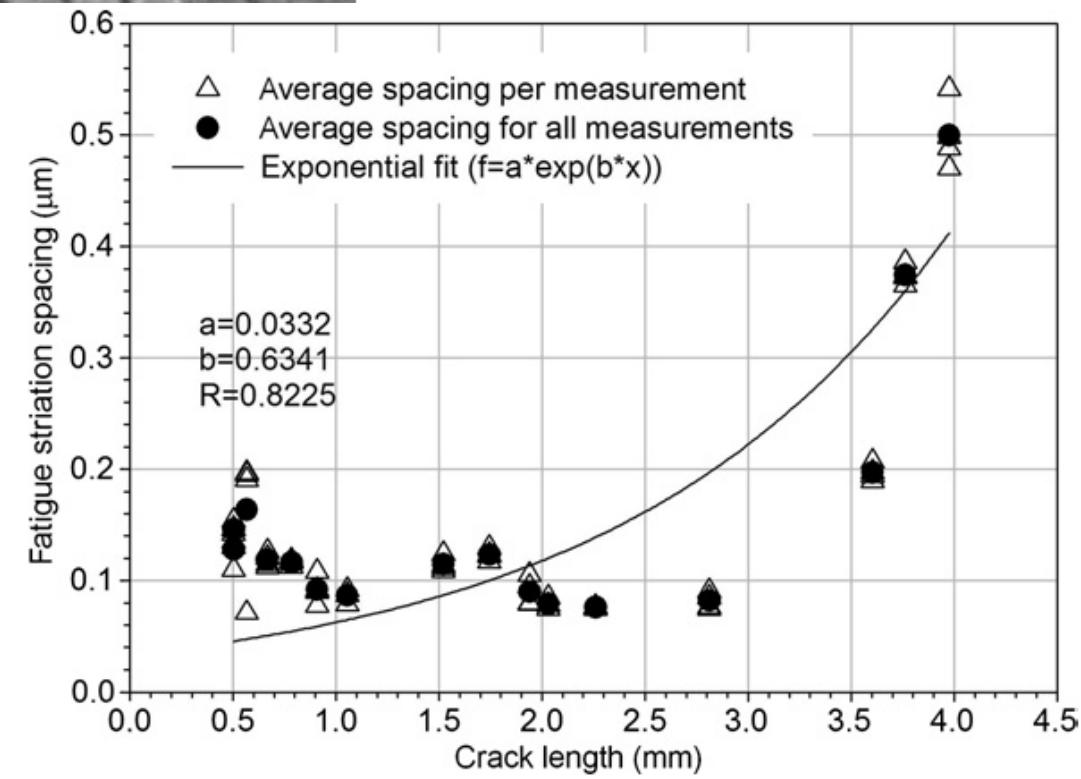
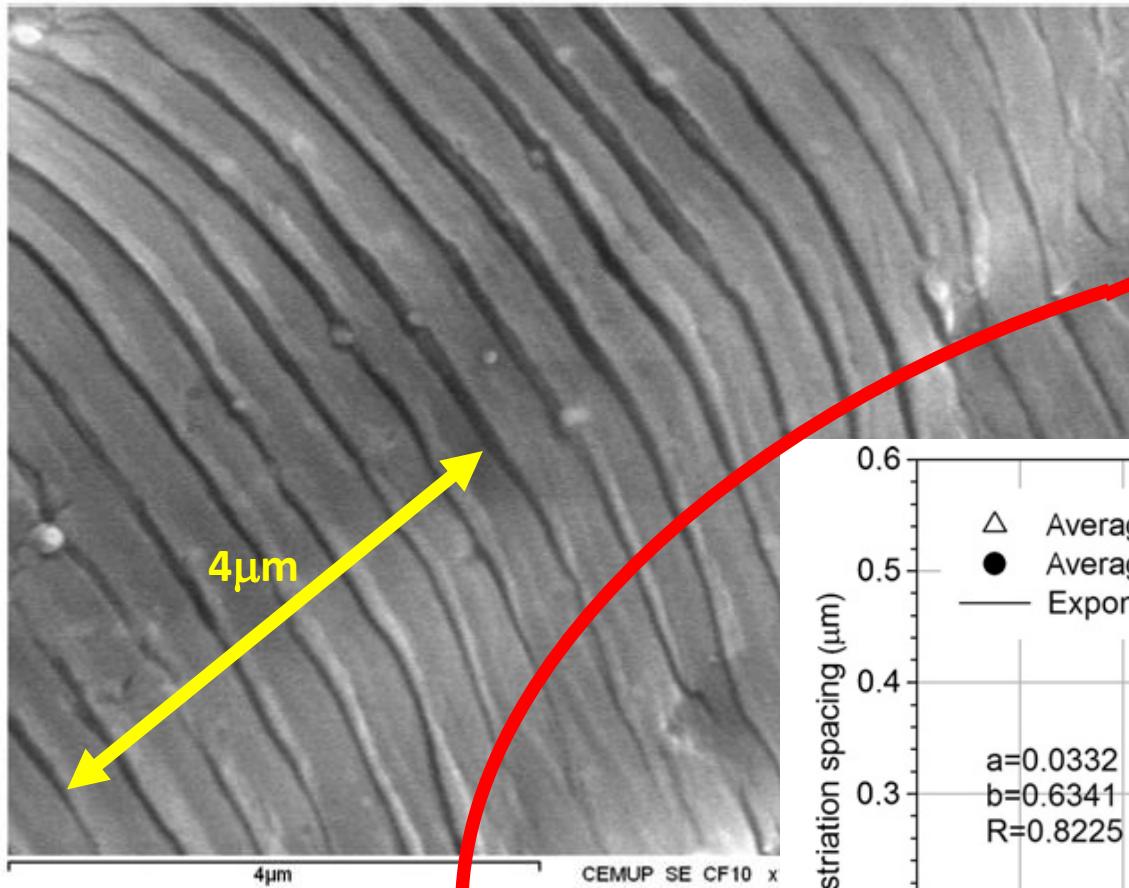


Fig. 8. Fatigue striation spacing *vs.* crack length for specimen of Al6082-T6.



estria:  $4 \mu\text{m} / 8 \approx 0.5 \mu\text{m}$

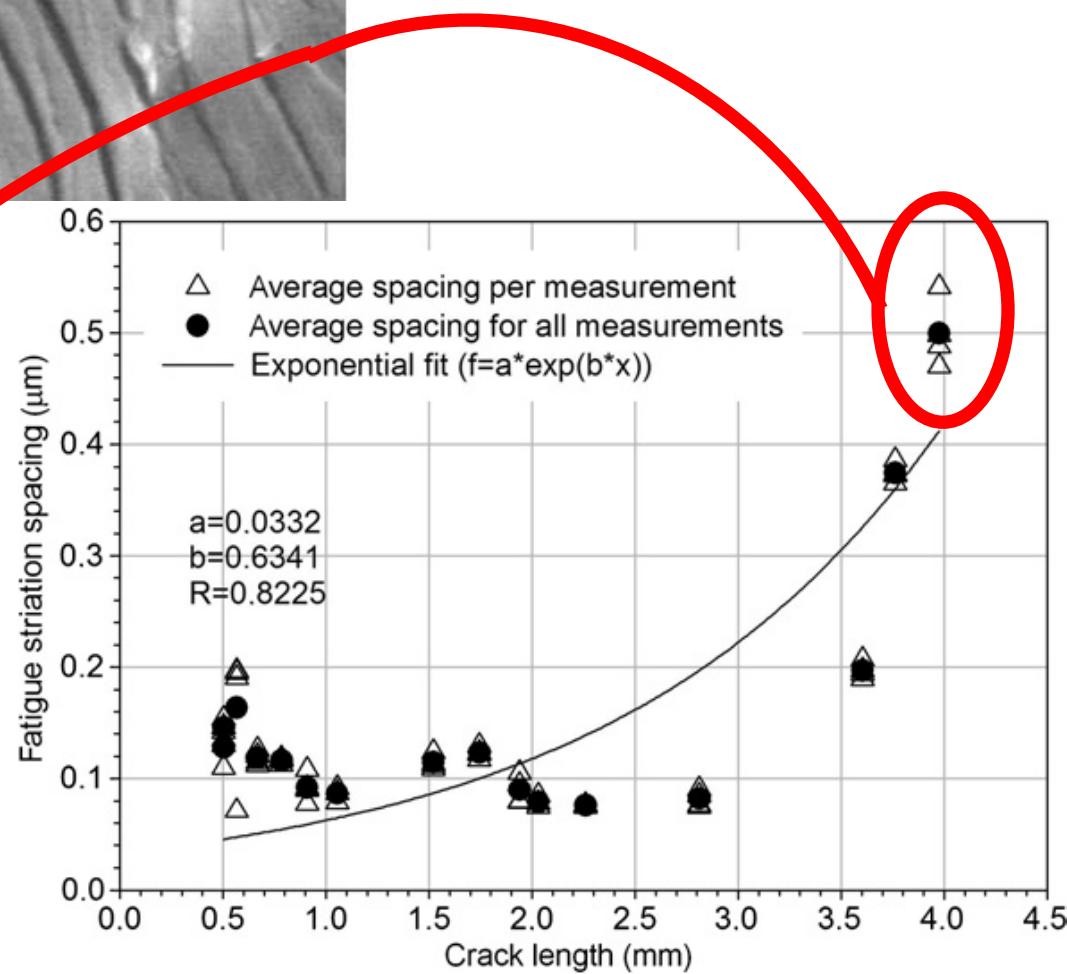


Fig. 8. Fatigue striation spacing *vs.* crack length for specimen of Al6082-T6.

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## **Introdução – exs. de casos**

- Haste
- Ligação soldada

## **Referência a conceitos básicos**

- Bibliografia de autores do DEMec da FEUP

Propagação de fendas

- Expansão de furos

Propagação de fendas em modo misto

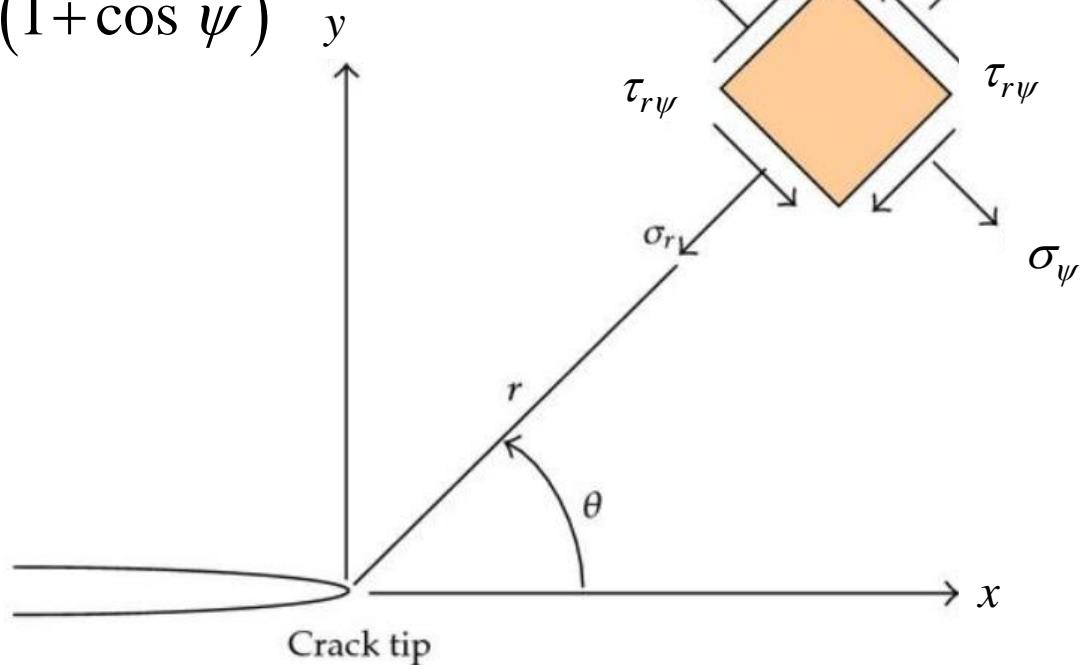
- O caso da flexão em 4 pontos

Métodos numéricos – o XFEM

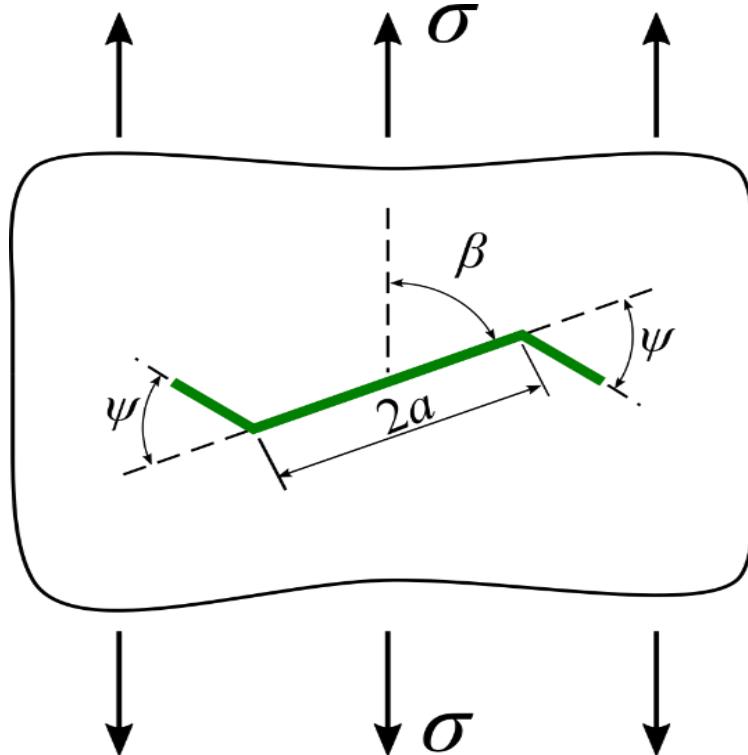
$$\sigma_r = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\psi}{2} \left( 3 - \cos \psi \right)$$

$$\sigma_\psi = \frac{K_I}{\sqrt{2\pi r}} \frac{1}{2} \cos \frac{\psi}{2} \left( 1 + \cos \frac{\psi}{2} \right)$$

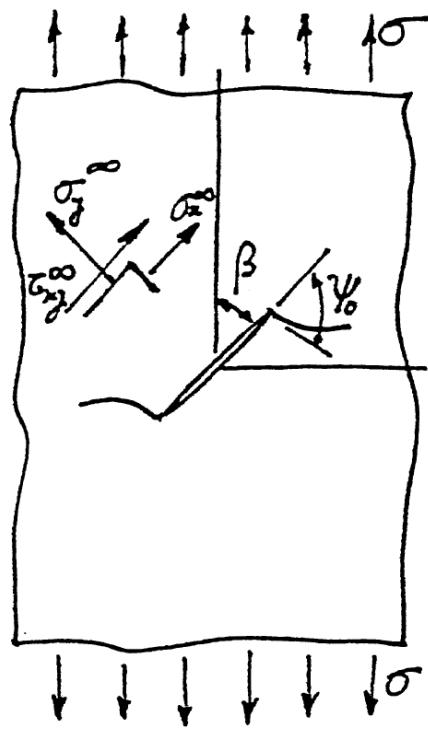
$$\tau_{r\psi} = \frac{K_I}{\sqrt{2\pi r}} \frac{1}{2} \sin \frac{\psi}{2} \left( 1 + \cos \psi \right)$$



Notation used in crack propagation direction studies from an initial pre-crack. In this figure  $\sigma$  it is the remote load applied,  $a$  it is the semi crack length,  $\psi_0$  is the angle between the remote load direction and the crack plane, and  $\beta$  is the angle of crack propagation in relation to the pre-crack direction.



Notation used in studies of the direction of crack propagation from an initial pre-crack of length  $2a$



$$\sigma_r = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\psi}{2} (3 - \cos \psi)$$

$$\sigma_\psi = \frac{K_I}{\sqrt{2\pi r}} \frac{1}{2} \cos \frac{\psi}{2} \left( 1 + \cos \frac{\psi}{2} \right)$$

$$\tau_{r\psi} = \frac{K_I}{\sqrt{2\pi r}} \frac{1}{2} \sin \frac{\psi}{2} (1 + \cos \psi)$$

recall that principal stresses are

$$\sigma_{1,2} = \frac{\sigma_r + \sigma_\psi}{2} + \sqrt{\left( \frac{\sigma_r - \sigma_\psi}{2} \right)^2 + \tau_{r\psi}^2}$$

$$\rightarrow \sigma_{1,2} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\psi}{2} \left( 1 \pm \sin \frac{\psi}{2} \right)$$

Based on the experimentally confirmed hypothesis that a crack propagates along a path perpendicular to the direction of the highest principal stress, (therefore the shear stress component in the expected crack propagation path is zero), Sih *et al.* obtained

$$\tau_{r\psi} = 0 = \frac{K_I}{\sqrt{2\pi r}} \frac{1}{2} \sin \frac{\psi}{2} (1 + \cos \psi) - \frac{K_{II}}{\sqrt{2\pi r}} \frac{1}{2} \cos \frac{\psi}{2} (1 - 3 \cos \psi)$$

$$\tau_{r\psi} = 0 = \frac{K_I}{\sqrt{2\pi r}} \frac{1}{2} \sin \frac{\psi}{2} (1 + \cos \psi) - \frac{K_{II}}{\sqrt{2\pi r}} \frac{1}{2} \cos \frac{\psi}{2} (1 - 3 \cos \psi)$$

$$0 = K_I \sin \frac{\psi}{2} (1 + \cos \psi) - K_{II} \cos \frac{\psi}{2} (1 - 3 \cos \psi)$$

$$0 = K_I \frac{\sin \frac{\psi}{2}}{\cos \frac{\psi}{2}} (1 + \cos \psi) - K_{II} \frac{\cos \frac{\psi}{2}}{\cos \frac{\psi}{2}} (1 - 3 \cos \psi)$$

$$0 = K_I \tan \frac{\psi}{2} (1 + \cos \psi) - K_{II} (1 - 3 \cos \psi)$$

but

$$\tan \frac{\psi}{2} = \sqrt{\frac{1 - \cos \psi}{1 + \cos \psi}} \rightarrow$$

$$0 = K_I \sqrt{\frac{1 - \cos \psi}{1 + \cos \psi}} (1 + \cos \psi) - K_{II} (1 - 3 \cos \psi) =$$

$$= K_I \sqrt{(1 - \cos \psi)(1 + \cos \psi)} - K_{II} (1 - 3 \cos \psi)$$

$$0 = K_I \sqrt{1 - \cos^2 \psi} + K_{II} (3 \cos \psi - 1)$$

$$K_I \sin \psi_o + K_{II} (3 \cos \psi_o - 1) = 0$$

in the present case,

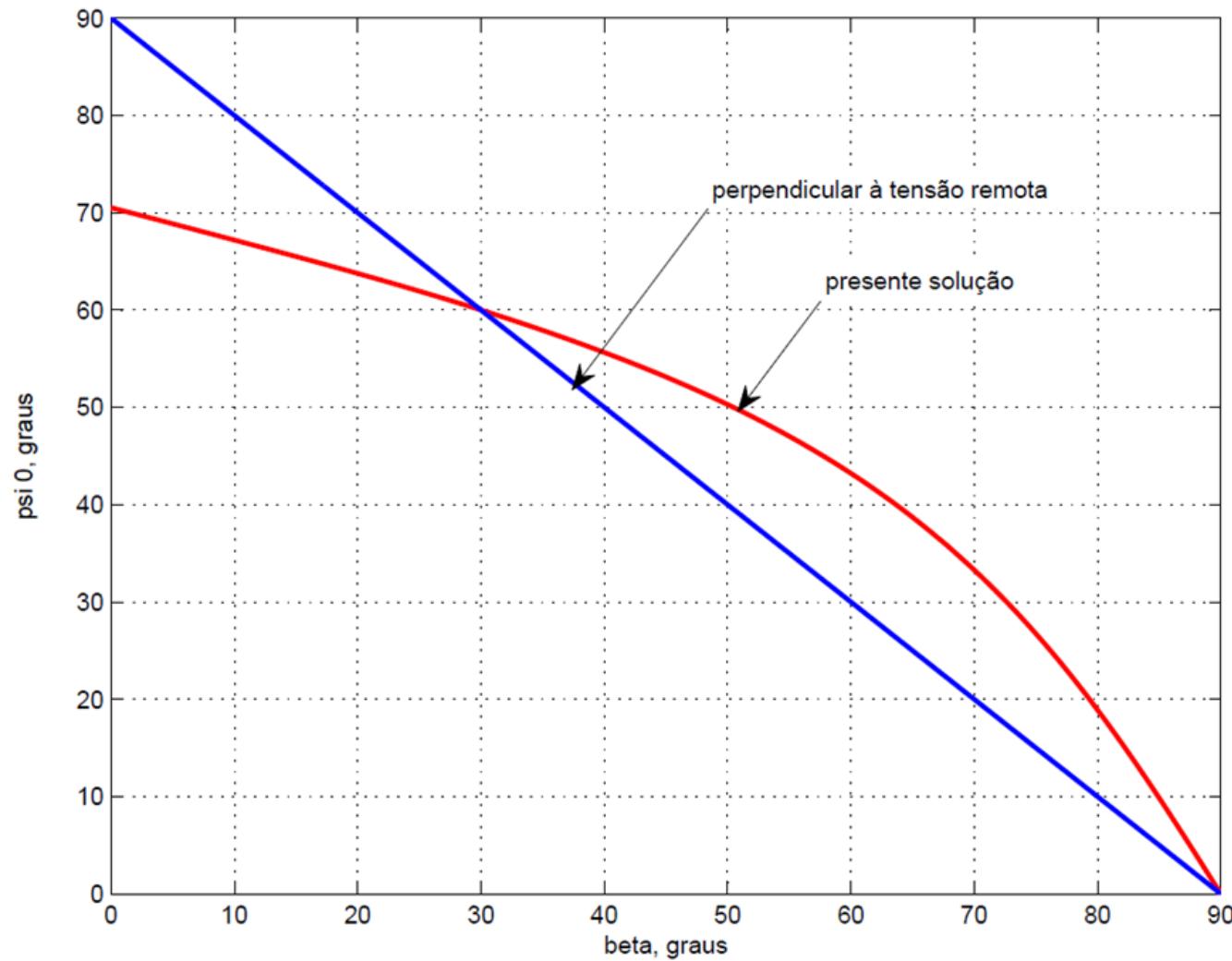
$$\sigma_y^\infty = \sigma \sin^2 \beta \quad ; \quad \sigma_x^\infty = \sigma \cos^2 \beta \quad ; \quad \tau_{xy}^\infty = \sigma \sin \beta \cos \beta$$

and

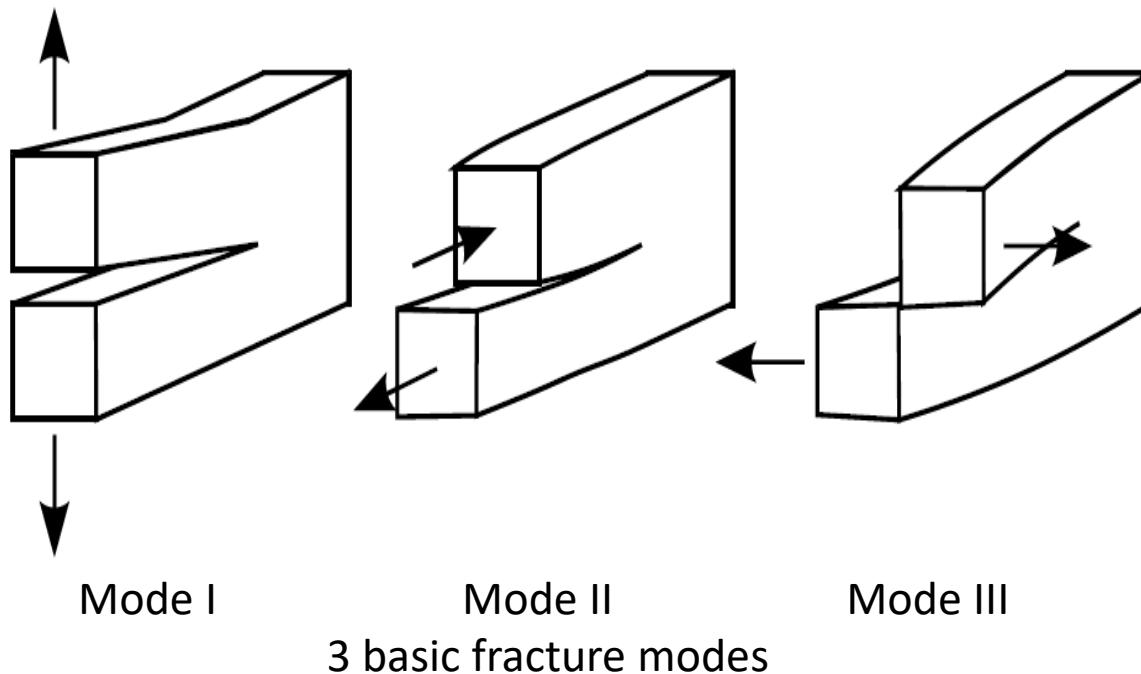
$$K_I = \sigma \sin^2 \beta \sqrt{\pi a} \quad ; \quad K_{II} = \sigma \sin \beta \cos \beta \sqrt{\pi a}$$

$$\rightarrow \cot \beta = \frac{-\sin \psi_o}{3 \cos \psi_o - 1}$$

when  $0 < \beta < \frac{\pi}{2}$ ,  $\psi_o$  is negative.

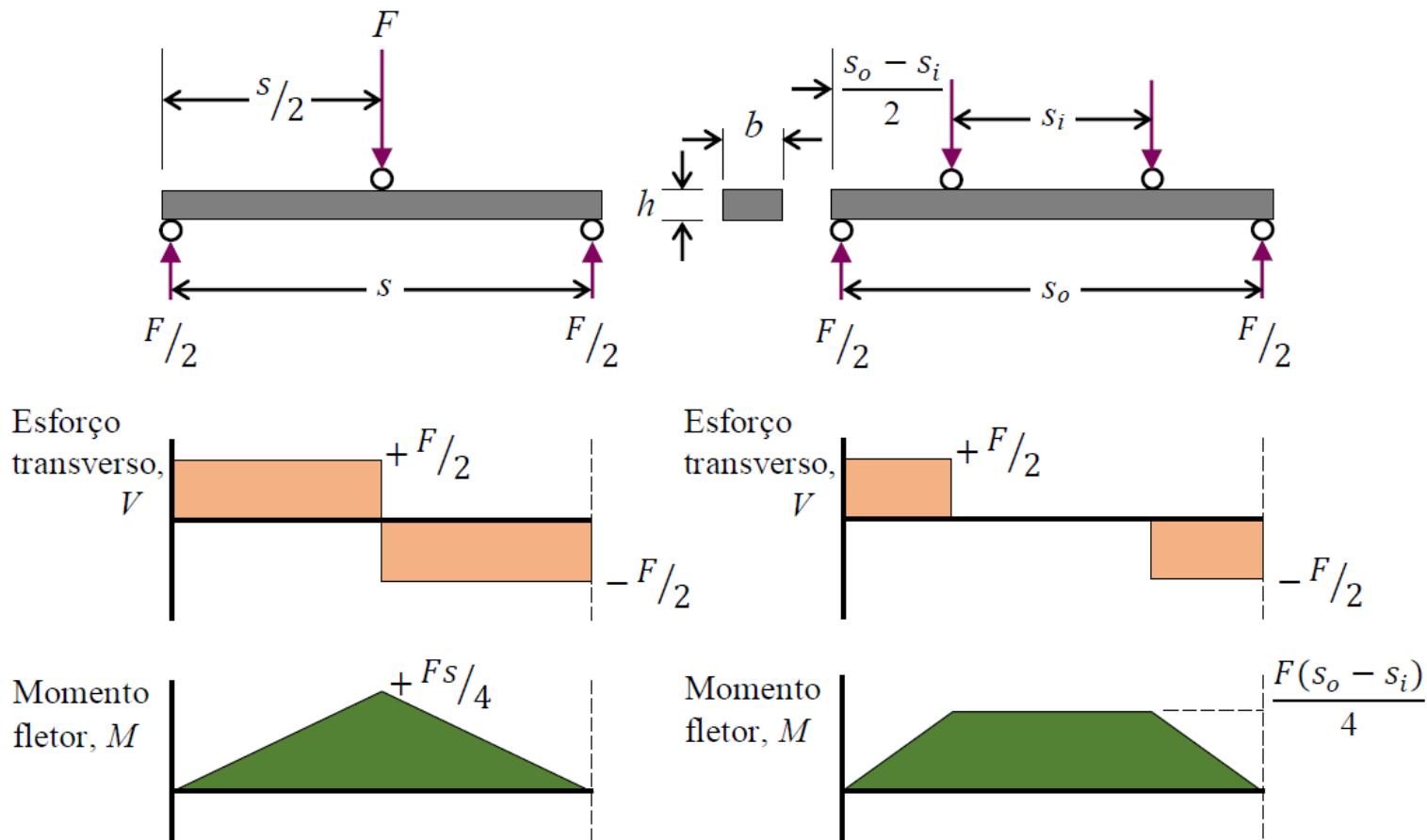


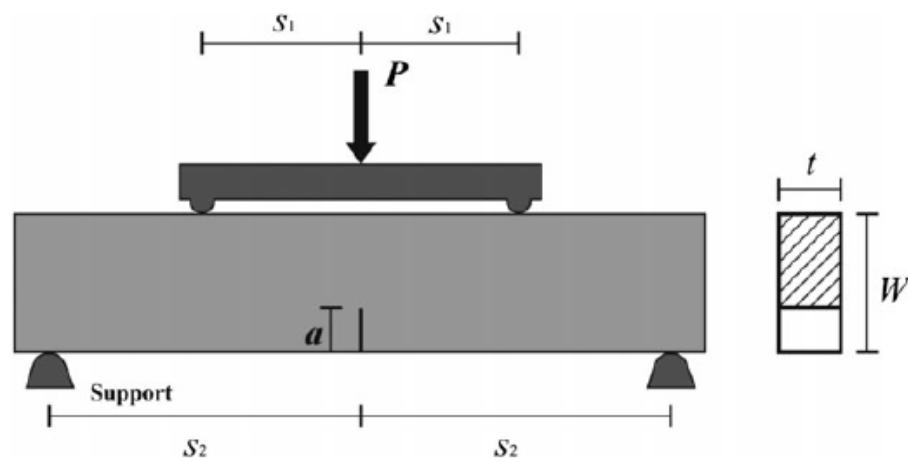
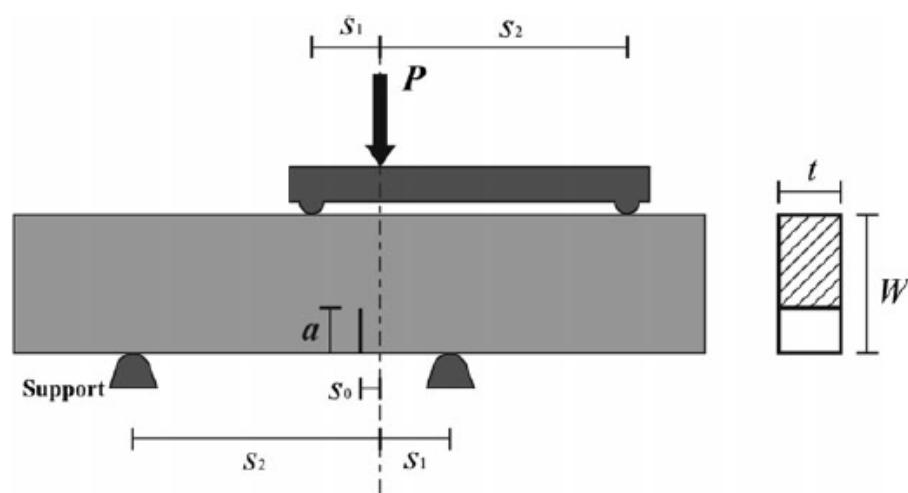
Relationship  $\psi_0$  versus  $\beta$ , figure of previous slide. Note  $\psi_0 < 0$ . The straight line represents the direction perpendicular to remote stress



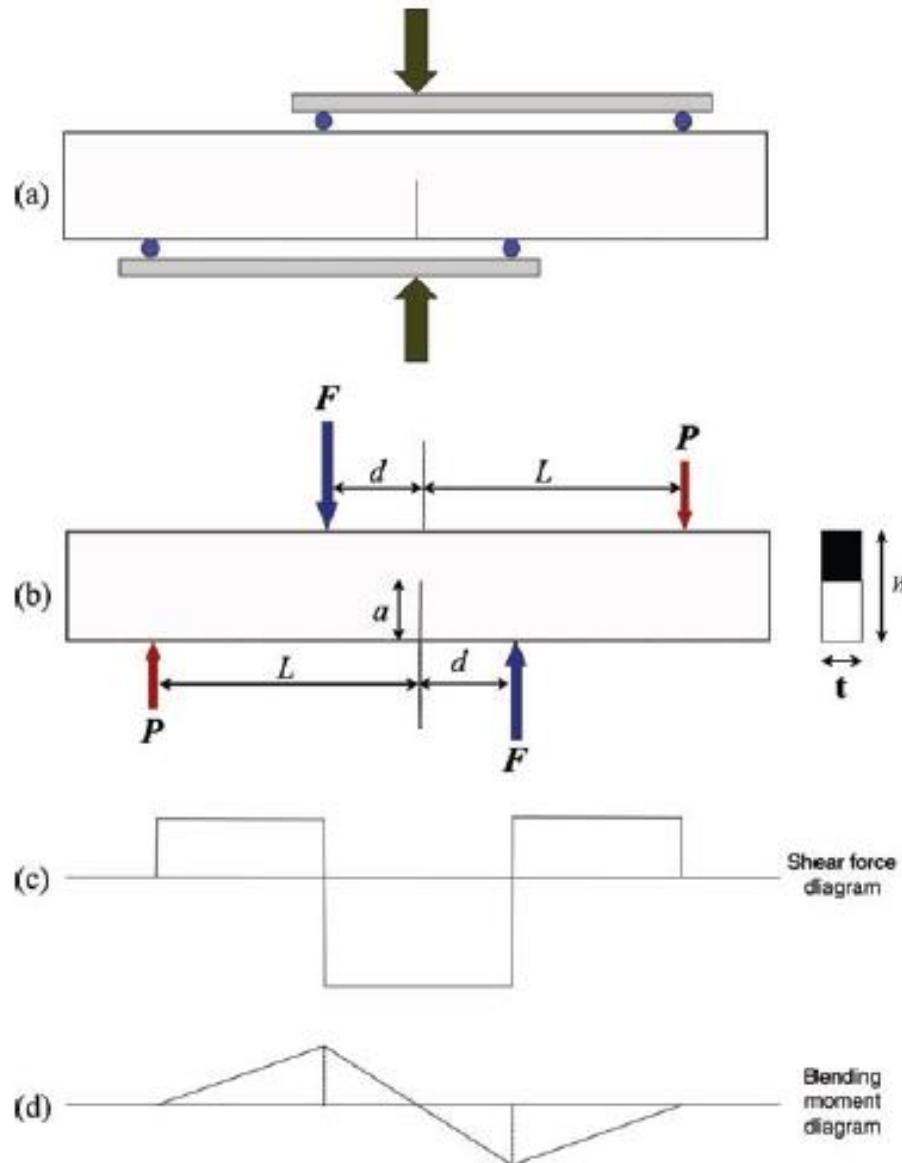
Most of the work on fatigue involves mode I situations, but in practice, mixed mode situations are often encountered.

bending tests are routine in characterizing the mechanical behavior of materials. The most common configuration is 3 point bending, which implies that the test takes place in the presence of shear or shear stress, and 4 point bending, eliminating shear stress

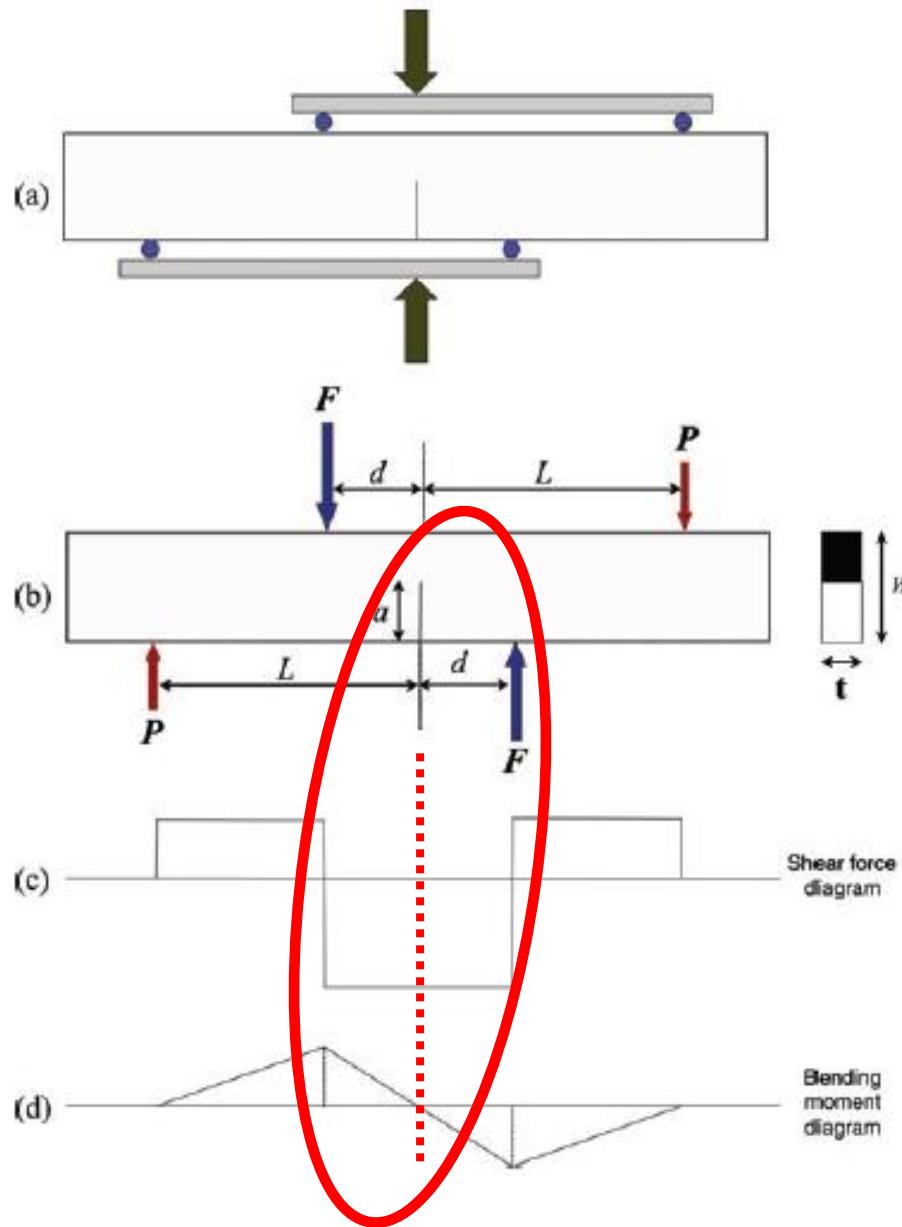




C. Wang, et al., *Fatigue and Fracture of Engineering Materials and Structures*, vol.39, pp.1193–1203, 2016



M.R. Ayatollahi, M.R.M. Aliha, Fatigue and Fracture of Engineering Materials and Structures,  
vol.34, (11), pp.898–907, 2011



M.R. Ayatollahi, M.R.M. Aliha, Fatigue and Fracture of Engineering Materials and Structures,  
vol.34, (11), pp.898–907, 2011

The last type of specimen mentioned (4-point bending specimen) allows to test a range of values of the mode I / mode II ratio (mixity value), and, in certain circumstances, allows evaluations of pure mode II (similarly to the Iosipescu notched specimen). Figure shows a notch machined in an AA6082 T6 Aluminum alloy specimen tested in 4-point bending, with cyclic loading and load ratio  $R=0$ . A pre-crack in mode I was initially made from the machined notch. The clearly visible sudden change in the direction of propagation was caused by the creation of a pure mode II situation, which in this type of test pieces is done changing the position of the load application points.



Example of deviation from the direction of propagation of a crack due to a change in the ratio I mode / mode II (mode mixity): detail of a flexion specimen in 4 points of AA 6082 T6 tested at FEUP by L. Gicquel, 2017

Work by Baganha Marques *et al.* at FEUP shows an AA6082 T6 4-point bending specimen. In one side of the test pieces several auxiliary markings of the test are visible. Again, the change in direction of propagation occurred when a pure mode II situation was created.



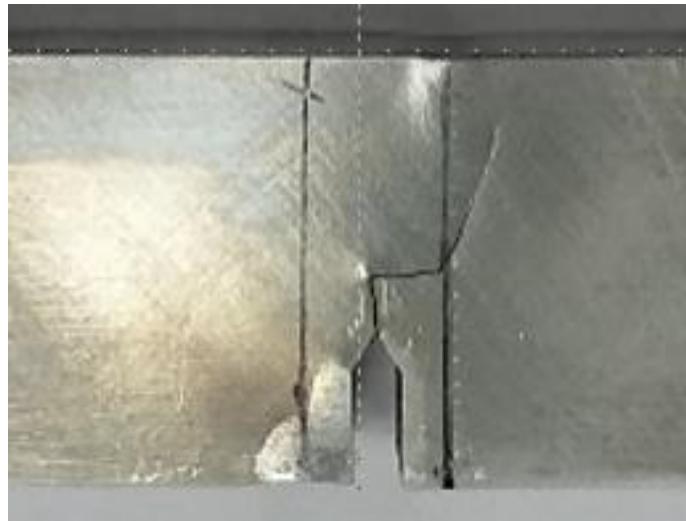
Example of deviation from the direction of propagation of a crack due to a change in the ratio of mode I / mode II: 4-point bending tests of AA 6082 T6, FEUP, J. Baganha Marques, 2017.

J. Baganha Marques, FEUP, 2018

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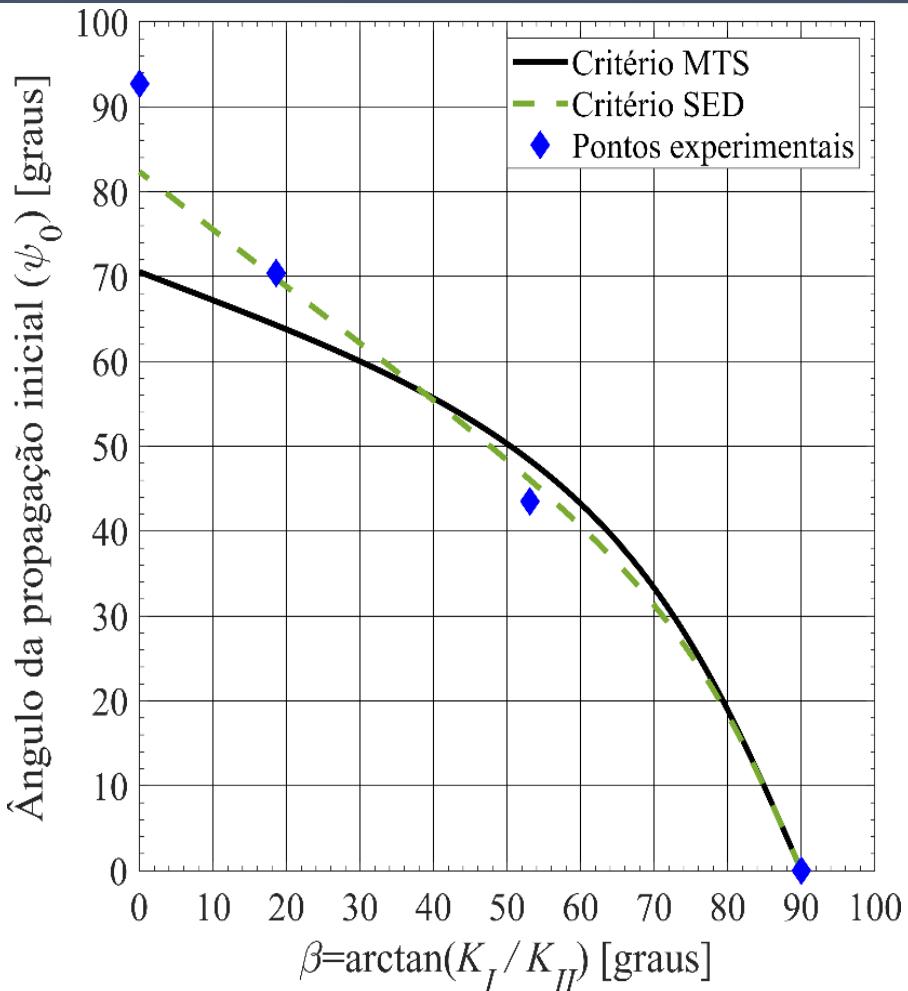


J. Baganha Marques, FEUP, 2018

Next Figure presents experimental values of the angle  $\psi_0$  as a function of the ratio  $K_I/K_{II}$ . The  $K_I/K_{II}$  ratio is related to the angle  $\beta$  of the basic situation (previous slide) through the relationship:

$$\beta = \arctan\left(\frac{K_I}{K_{II}}\right)$$

The experimental data was compared with theoretical predictions using the MTS and SED (strain energy density) criteria. The figure illustrates the good agreement between the prediction of the SED criterion for the angles of propagation direction at the onset of crack growth from the pre-crack ( $\psi_0$  versus  $\beta$ ).



J. Baganha Marques, FEUP, 2018



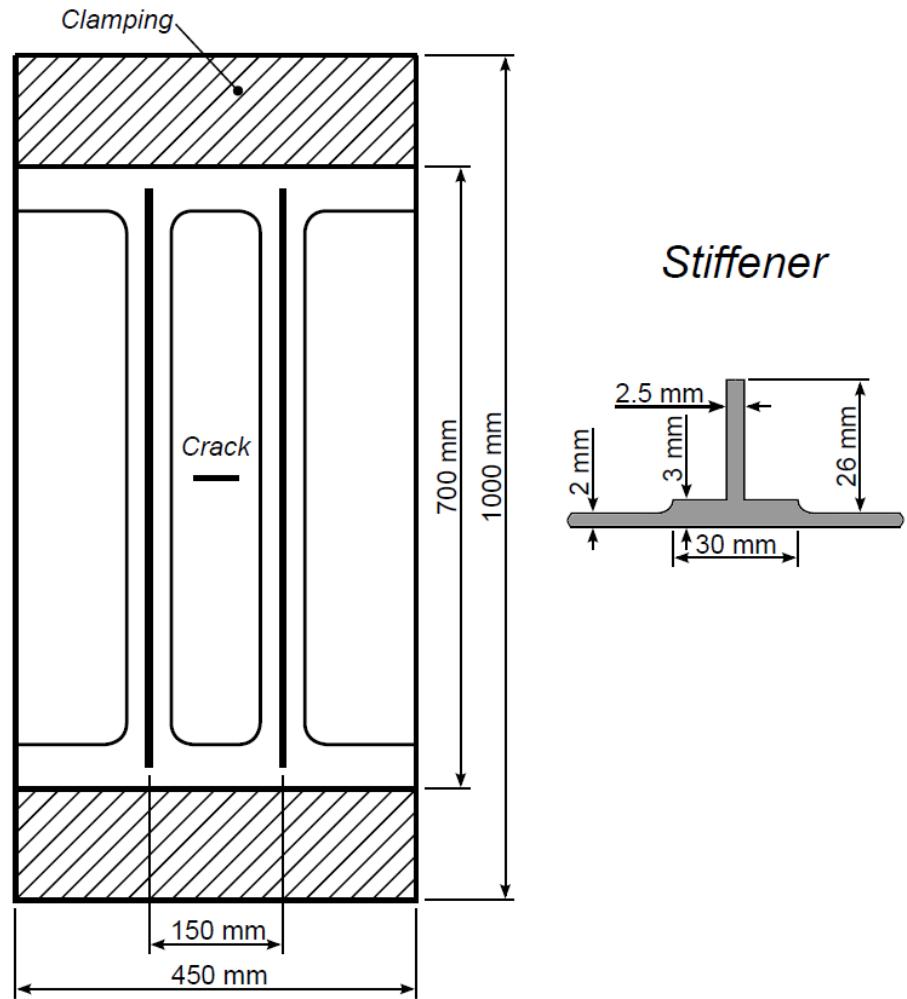
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Paulo C. M. Azevedo, 'Evaluation of the propagation of an inclined crack for the DaToN stiffened panel under uniaxial loading using 3D FE analysis', FEUP, 2008

## 1. Introduction and Procedure

The finite element (FE) method was used to predict the propagation of an inclined central through crack in the DaToN stiffened panel under uniaxial tensile loading. The 3D analyses were carried out using ABAQUS.

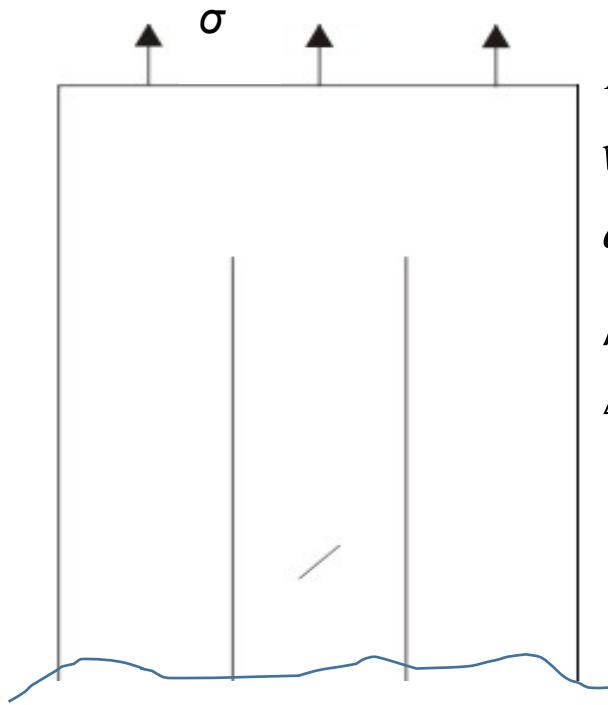
The stress intensity factors (SIFs) were determined using the  $J$  integral method. After obtaining  $K_I$  and  $K_{II}$  for the initial inclined crack, subsequent FE analyses were carried out for successive crack increments.



DaToN stiffened panel (specimen geometry)

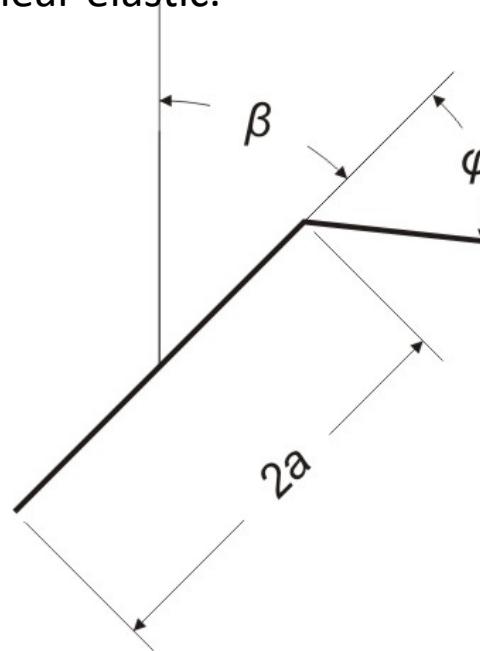
## 1. Introduction and Procedure

The direction of propagation was predicted based on  $K_I$  and  $K_{II}$ . The crack increment was the same for all the steps considered, and it was not small enough for this work to serve as a faithful prediction of the crack propagation behavior. All FE simulations are linear elastic.



Loaded panel with initial inclined crack

$$\begin{aligned}E_{Al} &= 70 \text{ GPa} \\ \nu &= 0.33 \\ a_0 &= 25 \text{ mm} \\ \beta_0 &= 45^\circ \\ \Delta a &= 4 \text{ mm}\end{aligned}$$



$$K_I \sin \varphi + K_{II} (3 \cos \varphi - 1) = 0$$

Scheme of crack propagation;  
notation

---

## 1. Introduction and Procedure

The calculation of the number of cycles is performed considering a dynamic load, even though the FE analyses were static. Since these simulations are carried out for  $\sigma = \sigma_{\max} = 110$  MPa the resultant SIFs and  $R$  are used to determine  $\Delta K_{eq}$ .

$$\Delta N = \frac{\Delta a}{C(\Delta K)^m}$$

$$C = 1.371 \times 10^{-11}$$

$$m = 2.744$$

$$R = \frac{\sigma_{\min}}{\sigma_{\max}} = 0.1$$

$$\Delta K_{eq} = (1 - R) \times \left[ K_I \left( \cos \frac{\varphi}{2} \right)^3 - 3K_{II} \left( \cos \frac{\varphi}{2} \right)^2 \left( \sin \frac{\varphi}{2} \right) \right]$$

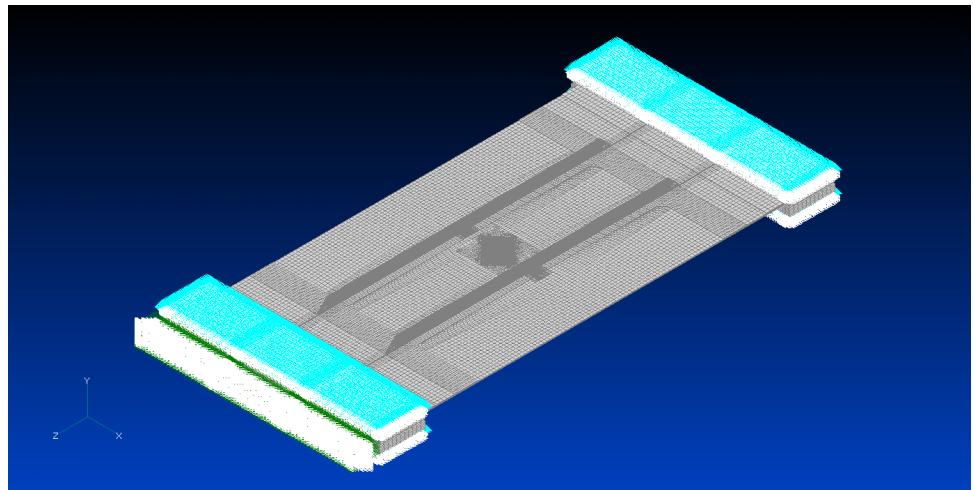
## 1. Introduction and Procedure

Four FE analyses were performed. The distance between each crack tip and the closer mid-side nodes is reduced to half of its original length, for all elements along the thickness of the panel.

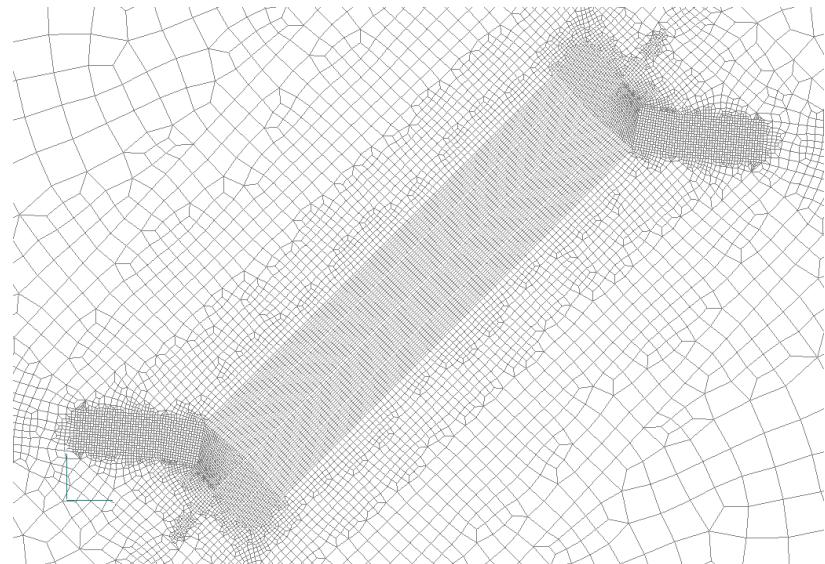
Five elements are defined along the thickness of the panel. Therefore, eleven nodes define each crack tip.

The value of  $\varphi$  that defines the direction of the crack increment is the average of all the angles determined for both crack tips, except for the ones that correspond to the surfaces of the panel.

$$E_{st} = 210 \text{ GPa} ; \quad \nu_{st} = 0.29$$

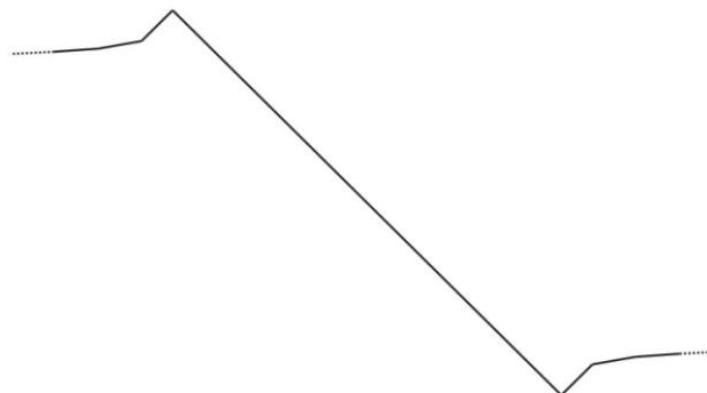


Meshed DaToN panel with load and constraints



Mesh detail: region of the crack

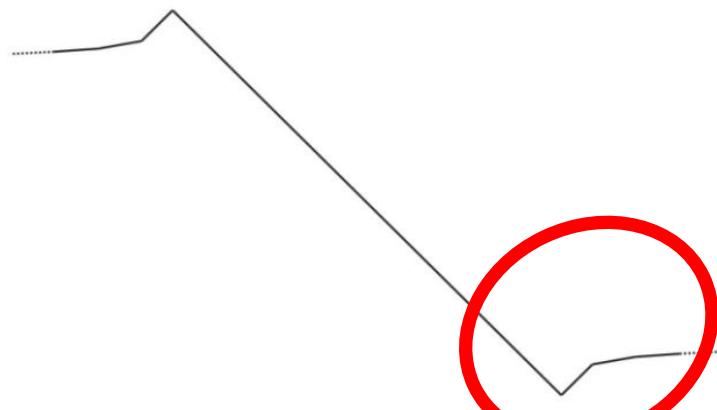
## 2. FE analysis results



Detail of crack propagation

Analysis	$\varphi$ (°)	$\Delta K_{eq}$ (Nmm <sup>-3/2</sup> )	$N$
1	90.5	358	2.9E+04
2	-35.0	999	1.7E+03
3	-6.0	1106	1.3E+03
4	-0.8	1176	1.1E+03

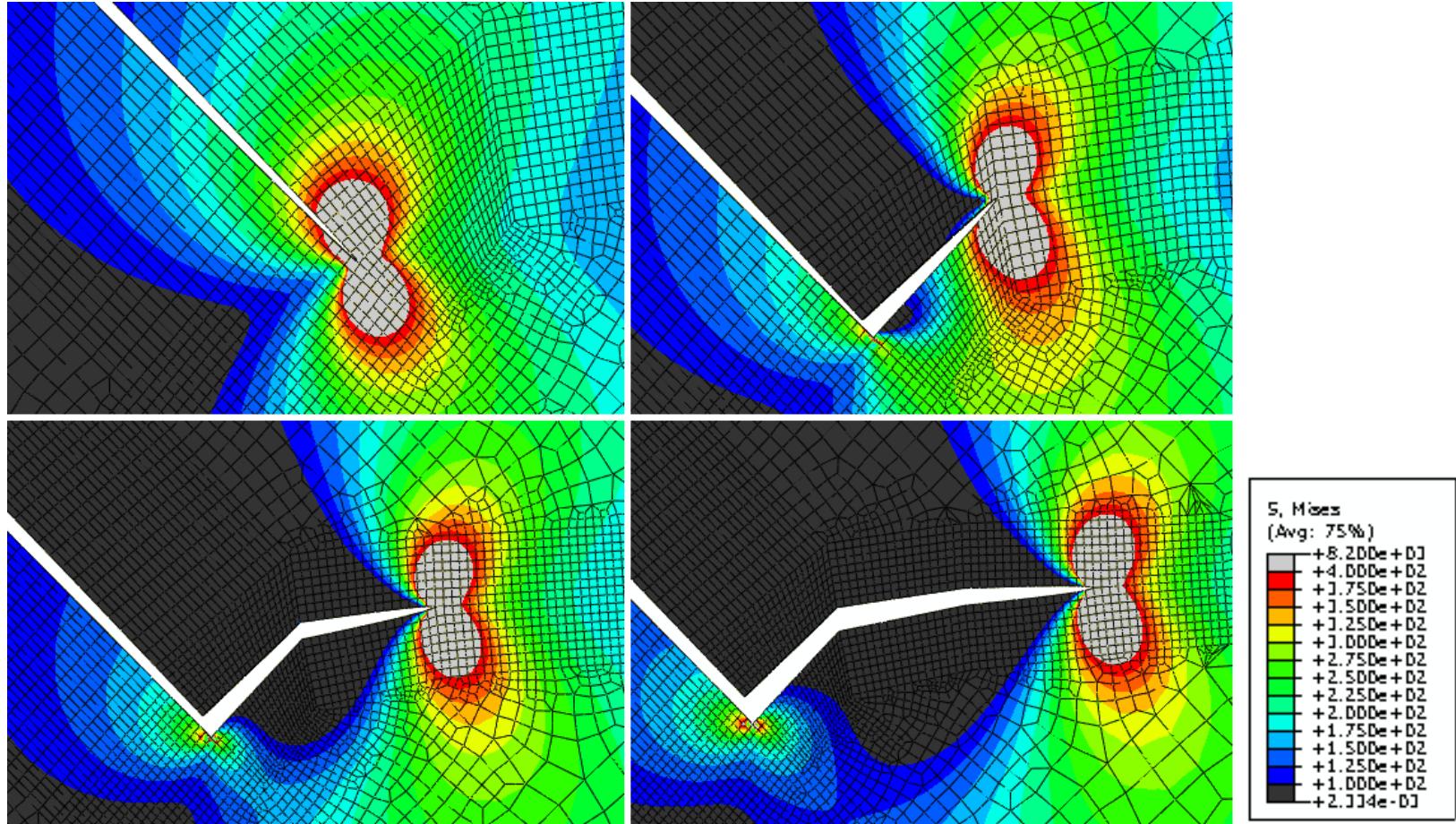
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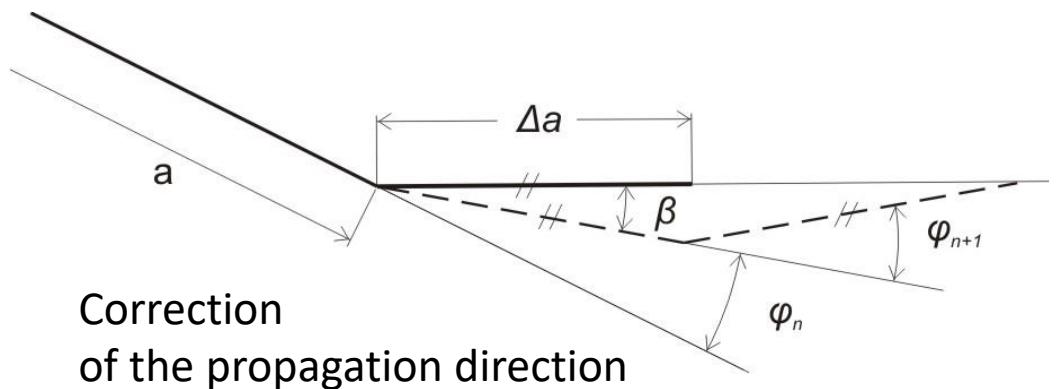
von Mises stress distribution after subsequent crack increments;  
zoom in the crack tip region

Paulo C. M. Azevedo, FEUP, 2008

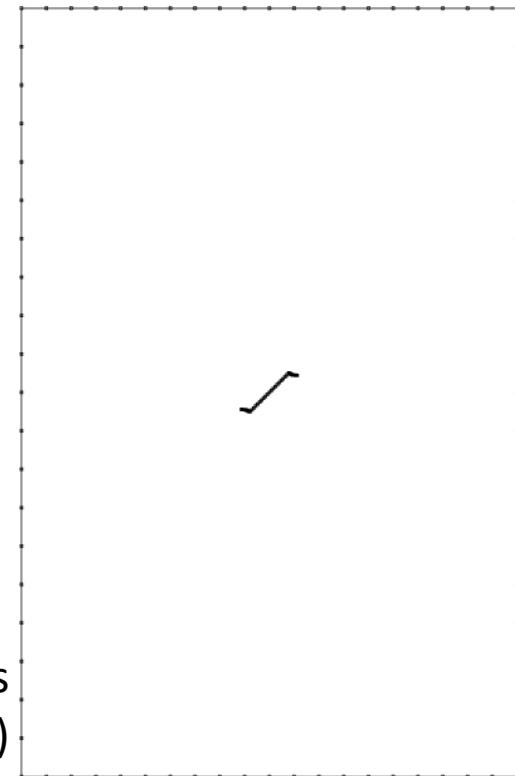
### 3. DBE analysis and correction of propagation direction

Dual Boundary Element (DBE) was used to predict the crack propagation direction for the same problem. The model used is bi-dimensional (constant thickness), and consequently does not include the stiffeners. A plane stress formulation is considered.

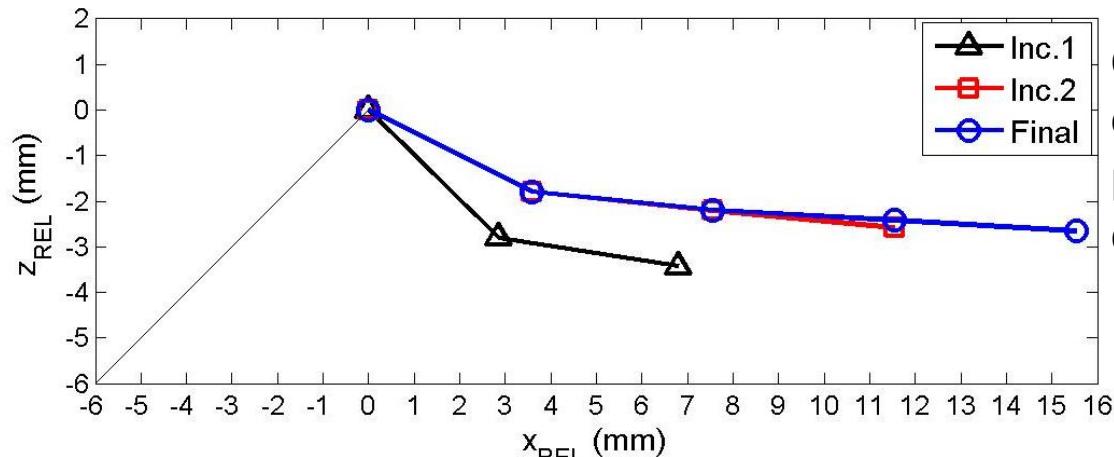
An alternative method, based on a simple technique to correct the crack propagation direction, was tested.



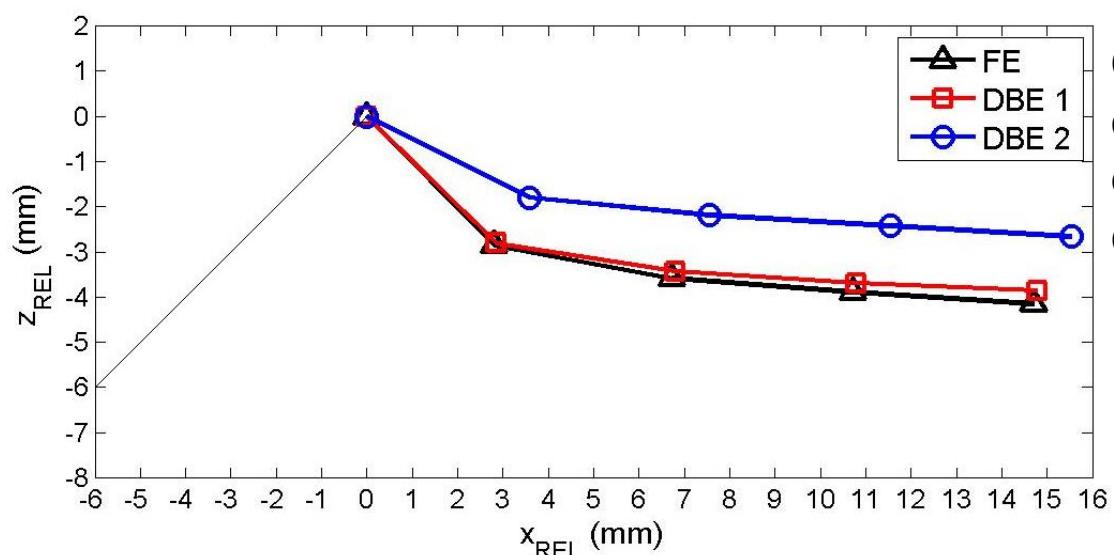
DBE mesh for third analysis  
(after two increments)



### 3. DBE analysis and correction of propagation direction



Crack path with  
correction of the  
propagation  
direction



Comparison of the  
crack paths  
obtained using the  
different methods

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## Concluding remarks

Under uniaxial loading, the mixed mode problem considered evolves to an almost pure mode I situation before the crack reaches the stiffeners.

The use of FEM required remeshing for crack growth modelling.

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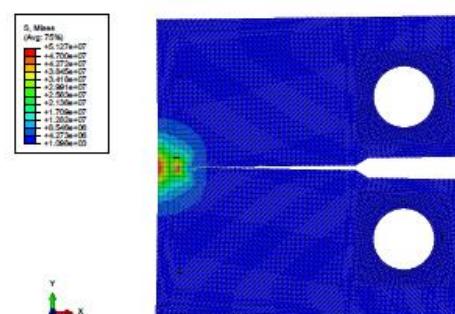
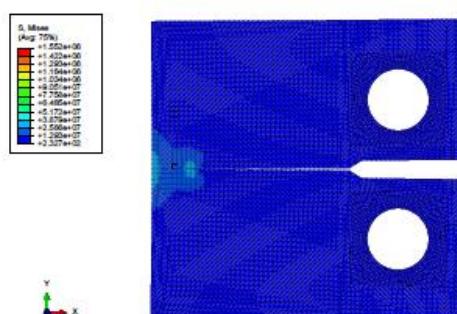
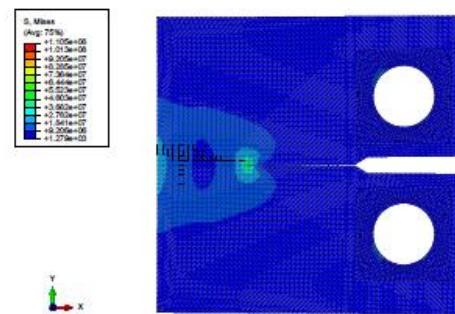
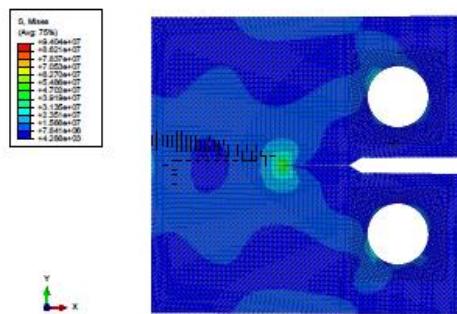
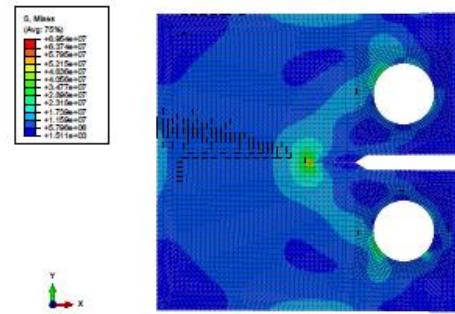
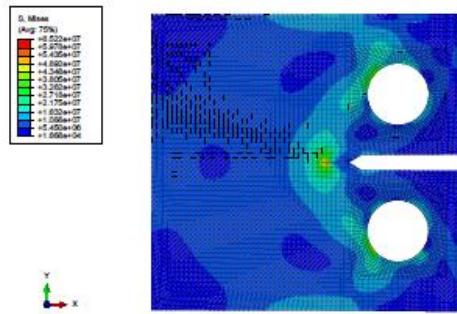
Propagação de fendas

- Expansão de furos

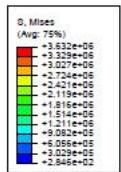
Propagação de fendas em modo misto

- O caso da flexão em 4 pontos

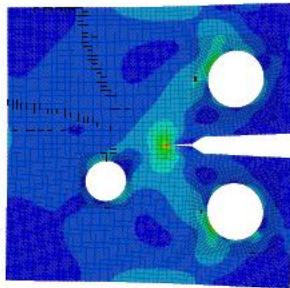
Métodos numéricos – o XFEM



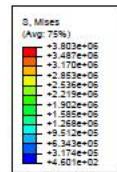
Maria Hermosilla, FEUP, 2016



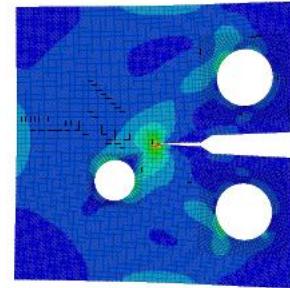
Y  
X



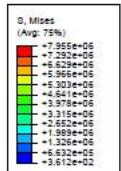
(a)



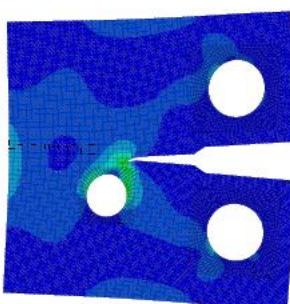
Y  
X



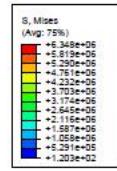
(b)



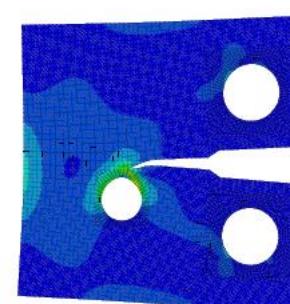
Y  
X



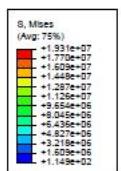
(c)



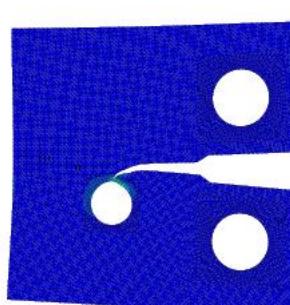
Y  
X



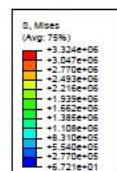
(d)



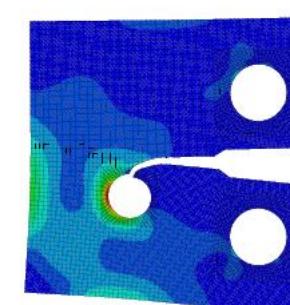
Y  
X



(e)

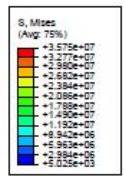


Y  
X

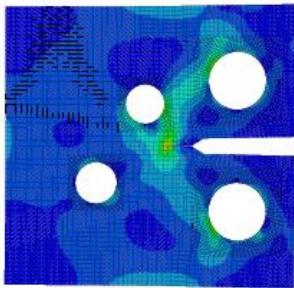


(f)

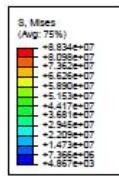
Maria Hermosilla,  
FEUP, 2016



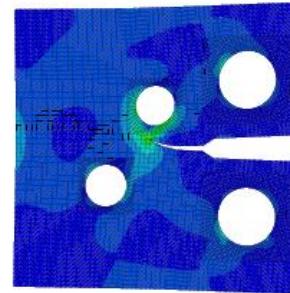
*x*  
*y*



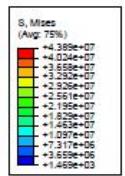
(a)



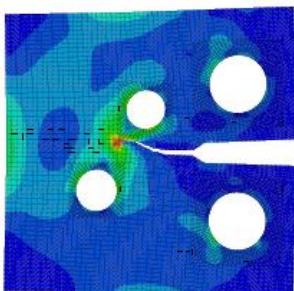
*x*  
*y*



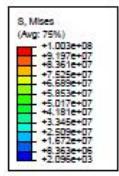
(b)



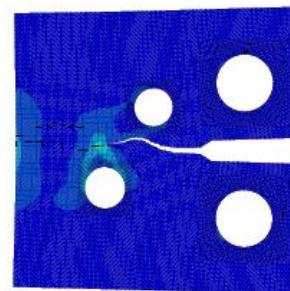
*x*  
*y*



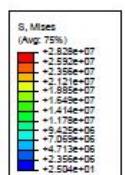
(c)



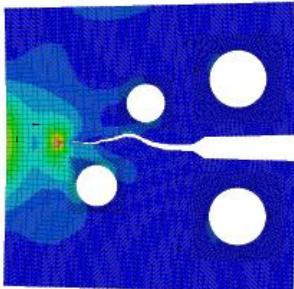
*x*  
*y*



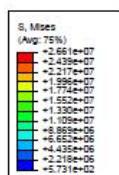
(d)



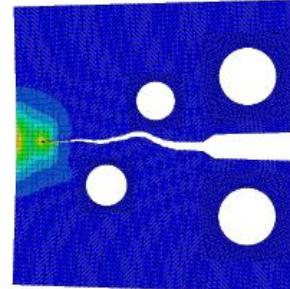
*x*  
*y*



(e)

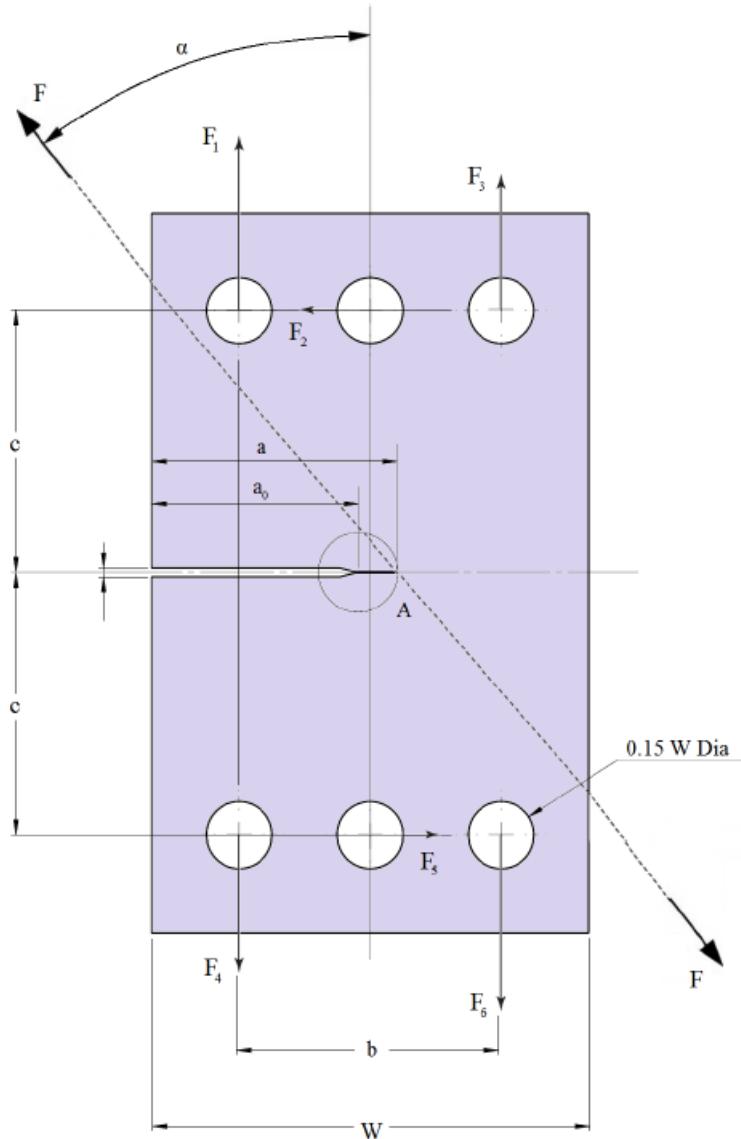


*x*  
*y*

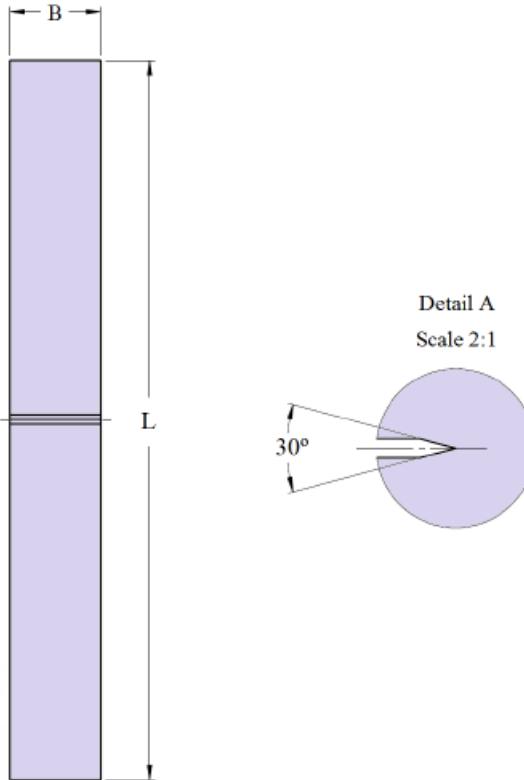


(f)

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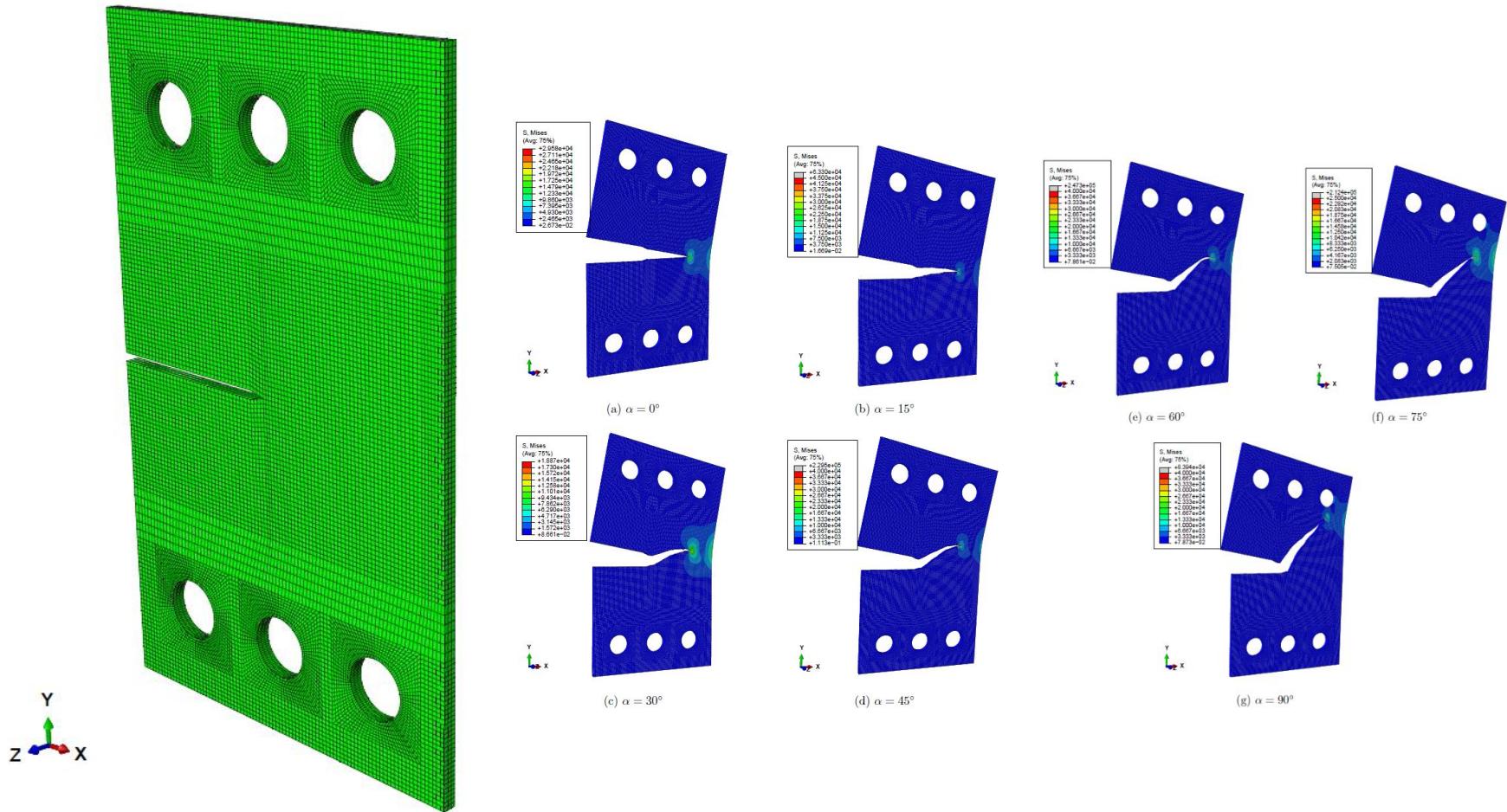


$$K_I = \frac{F\sqrt{\pi a}}{WB} \cos\alpha \sqrt{\frac{0.26 + 2.65(a/(W-a))}{1 - \frac{a}{W} \sqrt{1 + 0.55(a/(W-a))} - 0.08(a/(W-a))^2}}$$

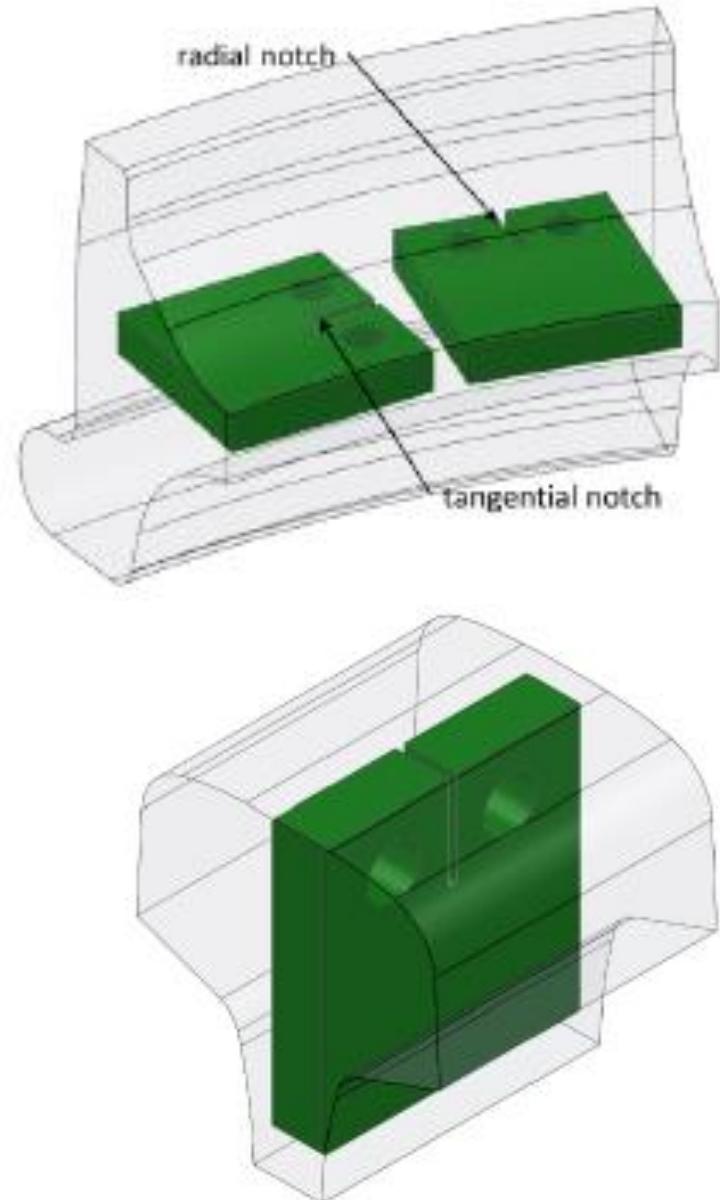


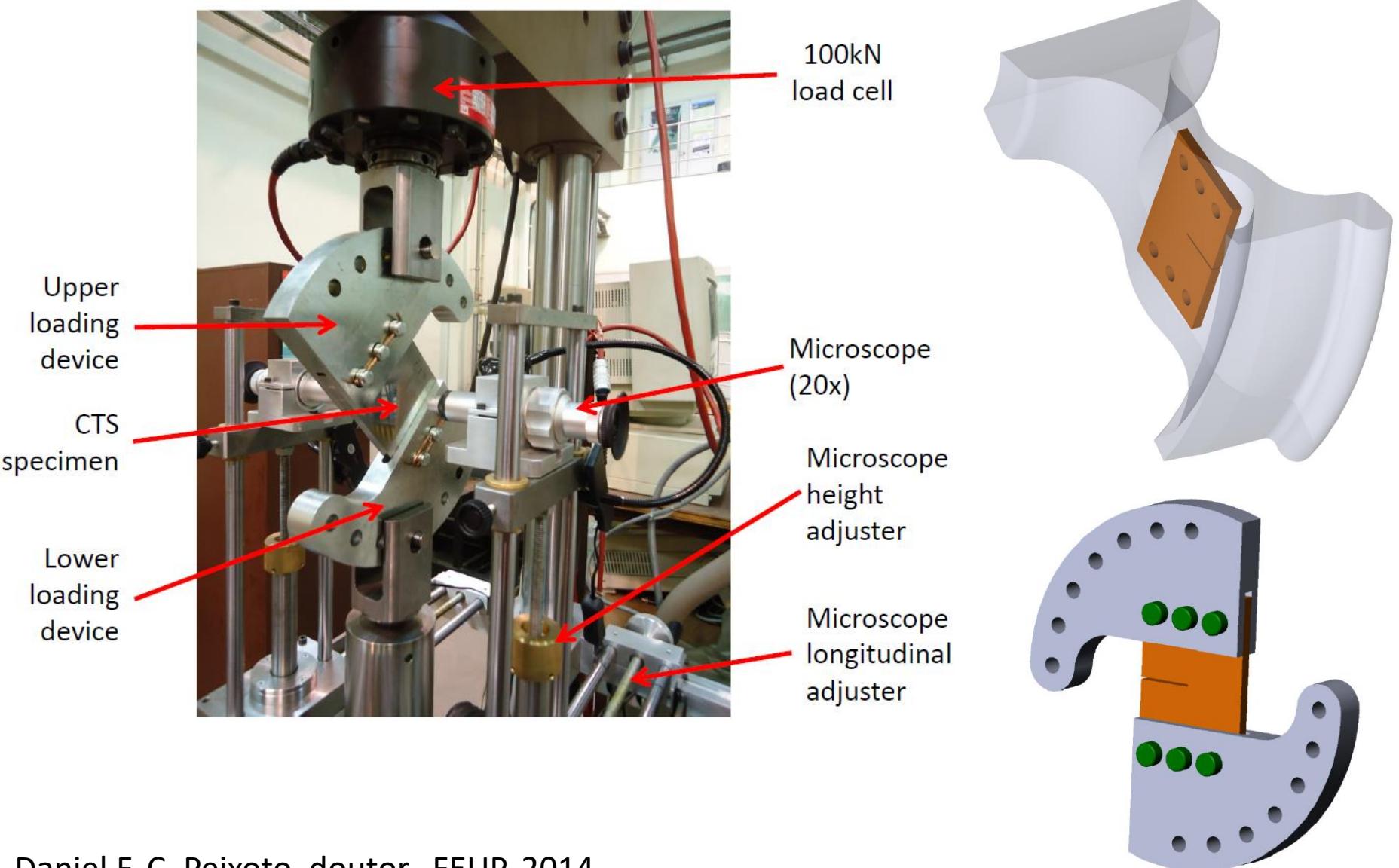
Maria Hermosilla,  
FEUP, 2016

$$K_{II} = \frac{F\sqrt{\pi a}}{WB} \sin\alpha \sqrt{\frac{-0.23 + 1.40(a/(W-a))}{1 - 0.67(a/(W-a)) + 2.08(a/(W-a))^2}}$$



Maria Hermosilla, FEUP, 2016

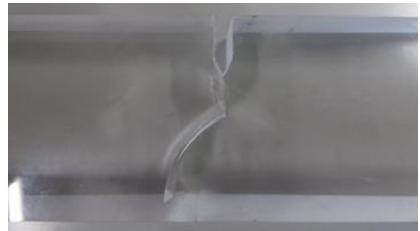




Daniel F. C. Peixoto, doutor., FEUP, 2014



## 4-point bending



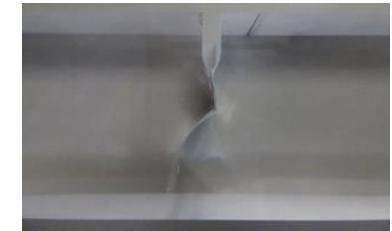
$$K_I/K_{II} = 0$$



$$K_I/K_{II} = 0,18$$



$$K_I/K_{II} = 0,234$$



$$K_I/K_{II} = 0,540$$



$$K_I/K_{II} = 0,899$$



$$K_I/K_{II} = 0,932$$



$$K_I/K_{II} = 1,799$$



$$K_I/K_{II} = 1,857$$



$$K_I/K_{II} = 2,232$$

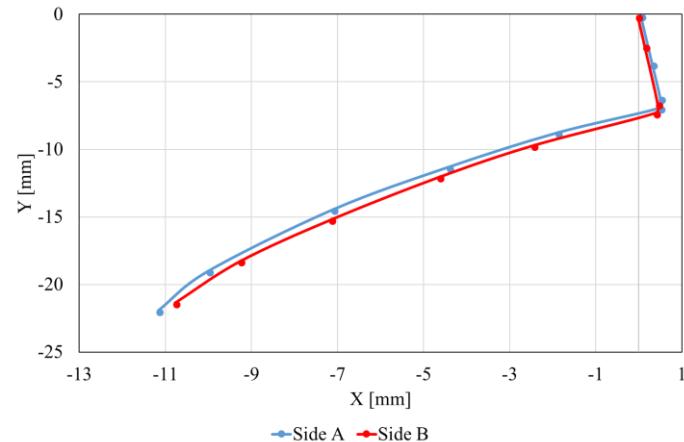


$$K_I/K_{II} = 2,811$$

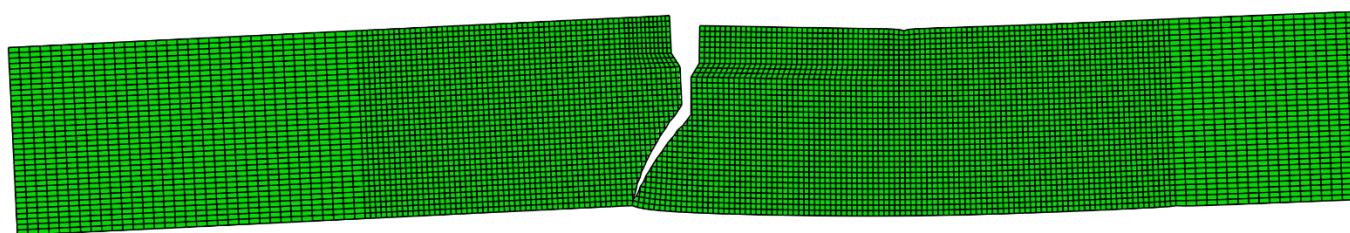


$$K_I/K_{II} = 3,749$$

## 2D crack path



Specimen 1, comparation between  
both sides



Specimen 8

