FEUP Universidade do Porto
Faculdade de Engenharia

Licenciatura em Engenharia Electrotécnica e de Computadores

EEC5275
Complementos de Comunicações Digitais

(2005-2006)

1st Mini-test

Duration: 1.30h (no books allowed) November 9th, 2005

1. Consider a noise reducing scheme of white noise $r(n)$ with power 10 and without DC component. Correlation between input signal $a(n)$ and the desired response $d(n) = r(n)$ is such that $a(n) = r(n) + 0.3r(n-1) + 0.1r(n-2)$. Samples of $a(n)$ are applied at the input of an N-tap transversal filter.

a) (2 pts.) Find the input autocorrelation matrix **R** and the crosscorrelation vector **p** when $N = 5$.

b) (2 pts.) Find the filter taps which minimize the mean square error (MSE) $\varepsilon = E[e^2(n)]$, when $N = 2$.

c) (2 pts.) Show that, when $N = 2$, the isometric curves of the performance surface ε drawn on the coordinated axes system (ν_0, ν_1) obey to a equation of the form $\frac{\nu_0^2}{a^2} + \frac{\nu_1^2}{b^2} = 1$, that is, they are ellipses centered at (0,0) with semi-axes a and b. Write down the expressions (not the values!) of a and b.

2. A signal is applied at the input of an adaptive filter. The eigenvalue and eigenvector matrices of its autocorrelation matrix **R** are, respectively,

$$\Lambda = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix} \qquad \mathbf{Q} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

Assume that $\mathbf{p} = \begin{bmatrix} 5 & 3 \end{bmatrix}^T$ and that the minimum mean square error is $\varepsilon_{min} = 0.6$. The steepest descent (or gradient) algorithm is to be used.

a) (2 pts.) Compute the desired response power.

b) (2 pts.) Find ε when $\mathbf{c}_e = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}^T$.

c) (1 pt.) Present a range for μ in order to maintain the algorithm's convergence.

d) (2 pts.) Determine the optimal step size, μ_{opt} (in the convergence speed sense).

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e) (1 pt.) Which natural mode, ν_0 or ν_1, does converge faster towards zero? Why?

f) (2 pts.) Now suppose $\mathbf{c}_e(0) = [-1.8 \quad 0.2]^T$ and $\mu = 0.15$. Write down an expression for ε, in terms of the n-th iteration, that shows ε approaching the minimum value ε_{min}. You may confirm that $\varepsilon(4) \approx 0.74$.

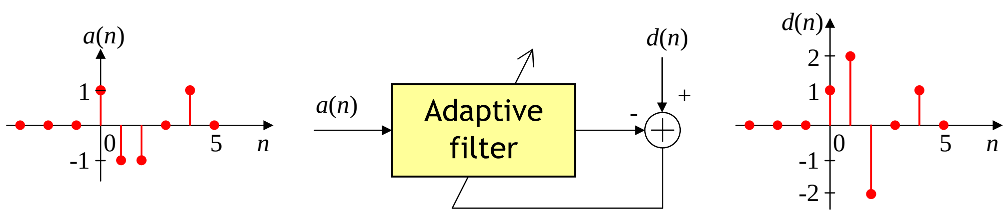
3. In some situations an adaptive algorithm is used to minimize the cost function $J(n) = E\left[e^2(n)\right] + \gamma \mathbf{c}^T(n)\mathbf{c}(n)$, where $0 < \gamma << 1$.

a) (2 pts.) Simplifying this algorithm the way it was done with the LMS algorithm, show that the update equation of the filter coefficient vector is given by $\mathbf{c}(n+1) = (1-2\mu\gamma)\mathbf{c}(n) + 2\mu e(n)\mathbf{a}(n)$

b) (2 pts.) Prove that, contrary to the LMS algorithm, now $E\left[\mathbf{c}(\infty)\right] \neq \mathbf{c}_{opt}$, i. e., this algorithm produces biased estimates.

c) (2 pts.) This algorithm is equivalent to adding white noise of variance γ to the input signal in the steepest descent and LMS algorithms. Using that reasoning find the widest range of values for the step size μ that assures convergence.

4. Consider the adaptive system of the figure. The filter has two taps initialized on zero and updated with an RLS algorithm using an unit forgetting factor (α = 1). Assume $\mathbf{P}(0) = 50\mathbf{I}$.



Please calculate:

a) (1 pt.) $\hat{\mathbf{R}}(1)$

b) (1 pt.) $\hat{\mathbf{p}}(2)$

c) (2 pts.) $\mathbf{P}(1)$

d) (1 pt.) $\mathbf{k}(1)$

e) (1 pt.) $e'(1)$

f) (1 pt.) $\mathbf{c}(1)$

The End