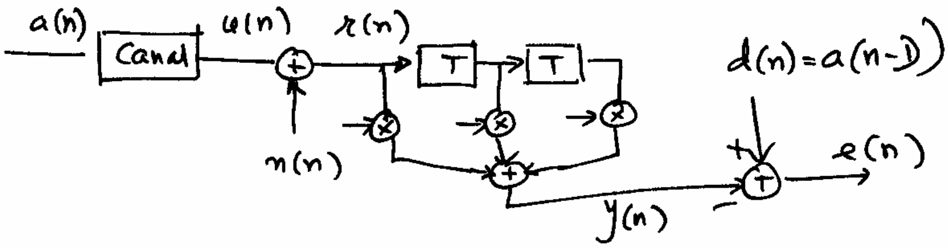
CCD (02-03) — 1º Mini-Teste (25-10-02) (1)

1.



Se $c_2 = \max(c_i) \Rightarrow$ o canal introduz um atraso de $2T$.
O igualizador introduz um atraso adicional de T segundos
$\Rightarrow$ atraso total : $3T \Rightarrow D = 3$, ou seja, $d(n) = a(n-3)$.

a) $R = R_u + R_n$; $R_n = \sigma_n^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$R_u = E \begin{bmatrix} u^2(n) & \cdots & \\ u(n)u(n-1) & u^2(n-1) & \\ u(n-2)u(n-1) & \cdots & \end{bmatrix}$$

Termo genérico : $E\left[u(n)\,u(n-k)\right] = E\Big\{\left[c_1 a(n-1) + c_2 a(n-2) + c_3 a(n-3)\right]$
$\times \left[c_1 a(n-1-k) + c_2 a(n-2-k) + c_3 a(n-3-k)\right]\Big\}$

k pode tomar os valores 0, 1 e 2. Atendendo a que os dados sucessivos $a(n)$ são independentes :

$k=0$: $E\left[u^2(n)\right] = E\left[u^2(n-1)\right] = E\left[u^2(n-2)\right] = c_1^2 + c_2^2 + c_3^2$

$k=1$ $E\left[u(n)u(n-1)\right] = E\left[u(n-1)\,u(n-2)\right] = c_1c_2 + c_2c_3$

$k=2$ $E\left[u(n)\,u(n-2)\right] = c_1c_3$

$$\Rightarrow R_u = \begin{bmatrix} c_1^2+c_2^2+c_3^2 & c_1c_2+c_2c_3 & c_1c_3 \\ c_1c_2+c_2c_3 & c_1^2+c_2^2+c_3^2 & c_1c_2+c_2c_3 \\ c_1c_3 & c_1c_2+c_2c_3 & c_1^2+c_2^2+c_3^2 \end{bmatrix}$$

$$R = R_u + R_n = \cdots$$

$$\underline{p} = E\left[d(n)\underline{r}(n)\right] = E\left\{d(n)\left[\underline{u}(n)+\underline{n}(n)\right]\right\} = E\left[d(n)\underline{u}(n)\right]$$

$$d(n)\,\underline{u}(n) = d(n)\left[u(n) \quad u(n-1) \quad u(n-2)\right]^T$$

$$\Rightarrow \underline{p} = \begin{bmatrix} p_0 \\ p_1 \\ p_2 \end{bmatrix} \quad e \quad p_k = E\Big[d(n)\big[c_1 a(n-1-k) + c_2 a(n-2-k) + + c_3 a(n-3-k)\big]\Big] \quad , \; k=0,1 \text{ e } 2.$$

Ora $d(n) = a(n-D) = a(n-3)$, logo

$$\begin{array}{ll} k=0 & p_0 = c_3 \\ k=1 & p_1 = c_2 \\ k=2 & p_2 = c_1 \end{array} \quad \Longrightarrow \quad \underline{p} = \begin{bmatrix} c_3 \\ c_2 \\ c_1 \end{bmatrix}$$

b) $$R = \begin{bmatrix} 0{,}821 & 0{,}48 & 0{,}09 \\ 0{,}48 & 0{,}821 & 0{,}48 \\ 0{,}09 & 0{,}48 & 0{,}821 \end{bmatrix} \qquad \underline{p} = \begin{bmatrix} 0{,}3 \\ 0{,}8 \\ 0{,}3 \end{bmatrix}$$

$$\underline{c}_{opt} = R^{-1}\underline{p} = \left[-0{,}4796 \quad 1{,}5352 \quad -0{,}4796\right]^T \approx \left[0{,}48 \quad 1{,}54 \quad -0{,}48\right]^T$$

c) $$\varepsilon_{min} = \underbrace{E\left[d^2(n)\right]}_{E\left[a^2(n-3)\right] = 1} - \underline{p}^T\underline{c}_{opt} = 1 - \left[0{,}3 \quad 0{,}8 \quad 0{,}3\right]\begin{bmatrix} -0{,}4796 \\ 1{,}5352 \\ -0{,}4796 \end{bmatrix} = 0{,}0596$$

(c/ valores aprox. de c_{opt}: 0,056)

d) $\mu < \dfrac{1}{\lambda_{max}}$. Precisamos dos valores próprios:

$$\left|R - \lambda I\right| = 0 \Longrightarrow \lambda = \begin{cases} 0{,}1857 \\ 0{,}7310 \\ 1{,}5463 \end{cases} \Rightarrow \lambda_{max} = 1{,}5463$$

Valor máx.: $\mu_{max} = \dfrac{1}{\lambda_{max}} = 0{,}6467$

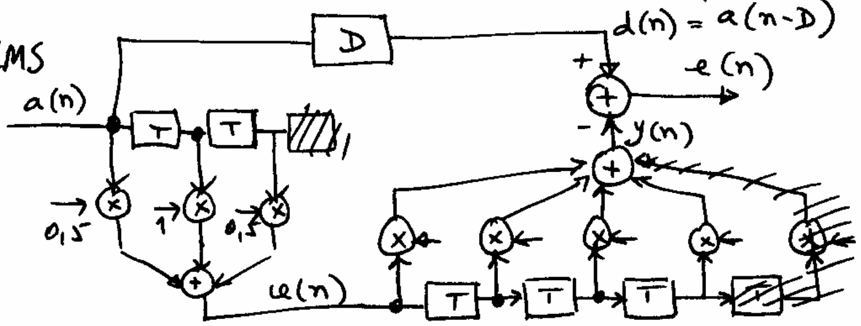
Margem de segurança: $\mu < \dfrac{1}{tr[R]} = \dfrac{1}{2{,}463} = 0{,}406$

e) $\underline{c}(n) - \underline{c}_{opt} = (I - 2\mu R)^n \left[\underbrace{\underline{c}(0)}_{0} - \underline{c}_{opt} \right]$

$$\Rightarrow \underline{c}(10) = \underline{c}_{opt} - (I - 2\mu R)^{10} \underline{c}_{opt} = [-0{,}1258 \quad 1{,}0005 \quad -0{,}1258]^T$$

(aproximado: $[-0{,}1253 \quad 1{,}004 \quad -0{,}1253]^T$).

2. LMS



Atraso total : $D = 3$; $\quad u(n) = 0{,}5\,a(n) + a(n-1) + 0{,}5\,a(n-2)$

$d(n) = a(n-3)$

Podemos escrever

n	$a(n)$	$d(n) = a(n-3)$	$u(n)$
⋮	−1	−1	−2
⋮	⋮	⋮	⋮
−2	−1	−1	−2
−1	−1	−1	−2
0	+1	−1	−1
1	+1	−1	+1
2	−1	−1	+1
3	+1	+1	0
⋮	⋮		

$$\underline{u}(n) = \begin{bmatrix} u(n) \\ u(n-1) \\ u(n-2) \\ u(n-3) \end{bmatrix}$$

a) Potência média de $d(n)$: $E[d^2(n)] = E[a^2(n-3)] = 1$.

b) $\underline{c}(n+1) = \underline{c}(n) + 2\mu\, e(n)\, \underline{u}(n)$

$e(n) = d(n) - \underline{c}^T(n)\, \underline{u}(n)$

$n = 0 \quad \underline{c}(0) = \underline{0}$

$d(0) = -1 \Rightarrow e(0) = -1; \quad \underline{u}(0) = [-1 \quad -2 \quad -2 \quad -2]^T$

$$\underline{c}(1) = \underline{c}(0) + 2\mu\, e(0)\, \underline{u}(0) = +2\mu\, [+1 \quad +2 \quad +2 \quad +2]^T = \begin{bmatrix} 0{,}04 \\ 0{,}08 \\ 0{,}08 \\ 0{,}08 \end{bmatrix}$$

$n = 1 \quad d(1) = -1$

$\underline{u}(1) = [1 \quad -1 \quad -2 \quad -2]^T \Rightarrow e(1) = d(1) - \underline{c}^T(1)\, \underline{u}(1) =$

$= 18\mu - 1 = 0{,}36 - 1 = -0{,}64$

$$\underline{c}(2) = +0{,}04 \begin{bmatrix} +1 \\ +2 \\ +2 \\ +2 \end{bmatrix} - 0{,}04 \times 0{,}64 \begin{bmatrix} 1 \\ -1 \\ -2 \\ -2 \end{bmatrix} = \begin{bmatrix} 0{,}0144 \\ 0{,}1056 \\ 0{,}1312 \\ 0{,}1312 \end{bmatrix}$$

c) Desajuste: $M \approx \mu\, tr[R]$.

$tr[R] \approx N E[u^2(n)]$. Ora $E[u^2(n)] = E\left[[0{,}5\,a(n) + a(n-1) + 0{,}5\,a(n-2)]^2 \right]$

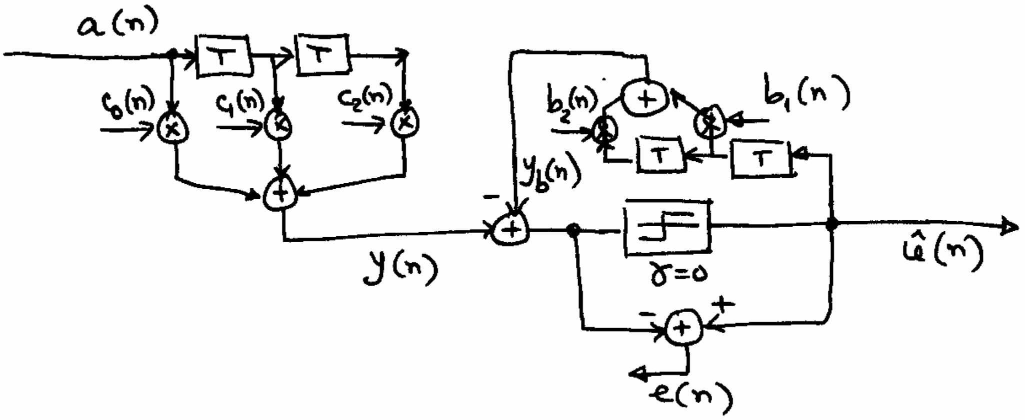
$= 0{,}25 \underbrace{E[a^2(n)]}_{1} + \underbrace{E[a^2(n-1)]}_{1} + 0{,}25 \underbrace{E[a^2(n-2)]}_{1} = 1{,}5$

Logo, $tr[R] \approx 4 \times 1{,}5 = 6$ e 4

$$M \approx \mu \, tr[R] = 0{,}02 \times 6 = 12\%$$

3. DFE

a) Diagrama de blocos



b)

	n					
	28	29	30	31	32	33
Entrada $a(n)$	0,25	−0,25	−0,5	0,1	0,8	−0,2
Saída $\hat{u}(n)$	+1	−1	−1	+1	+1	?

$\underline{c}(32) = [1 \;\; -2 \;\; 1]$, $\quad \underline{b}(32) = [0{,}7 \;\; 0{,}3]^T$, $e(32) = 1{,}3$

$$\text{LMS} \begin{cases} \underline{c}(n+1) = \underline{c}(n) + 2\mu\, e(n)\, \underline{a}(n) \\ \underline{b}(n+1) = \underline{b}(n) + 2\mu\, e(n)\, \underline{\hat{u}}(n-1) \end{cases}$$

$$e(n) = \hat{u}(n) - \underbrace{[y(n) - y_b(n)]}_{y_d(n)}, \qquad y_d(n) \underset{-1}{\overset{+1}{\gtrless}} \gamma = 0$$

Cálculo de $e(32)$:

$$\hat{u}(32) = +1$$

$$\left.\begin{aligned} y(32) &= \underline{c}^T(32)\, \underline{a}(32) = [1 \;\; -2 \;\; 1] \begin{bmatrix} 0{,}8 \\ 0{,}1 \\ -0{,}5 \end{bmatrix} = 0{,}1 \\ y_b(32) &= \underline{b}^T(32)\, \underline{\hat{u}}(31) = [0{,}7 \;\; 0{,}3] \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 0{,}4 \end{aligned}\right\} y_d(32) = -0{,}3$$

$$\Rightarrow e(32) = 1 - (-0{,}3) = 1{,}3$$

Cálculo dos coeficientes em $n=33$: (5)

$$\underline{c}(33) = \underline{c}(32) + 2\mu\, e(32)\, \underline{a}(32) = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + 2\mu \times 1{,}3 \begin{bmatrix} 0{,}8 \\ 0{,}1 \\ -0{,}5 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 + 2{,}08\mu \\ -2 + 0{,}26\mu \\ 1 - 1{,}3\mu \end{bmatrix}$$

$$\underline{b}(33) = \underline{b}(32) + 2\mu\, e(32)\, \hat{\underline{u}}(31) = \begin{bmatrix} 0{,}7 \\ 0{,}3 \end{bmatrix} + 2\mu \times 1{,}3 \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0{,}7 + 2{,}6\mu \\ 0{,}3 - 2{,}6\mu \end{bmatrix}$$

c)
$$y(33) = \underline{a}^T(33)\, \underline{c}(33) = \begin{bmatrix} -0{,}2 & 0{,}8 & 0{,}1 \end{bmatrix} \begin{bmatrix} \vdots \end{bmatrix} = -1{,}7 - 0{,}338\mu$$

$$y_b(33) = \hat{\underline{u}}^T(32)\, \underline{b}(33) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \vdots \end{bmatrix} = 1$$

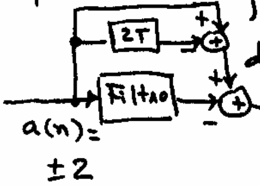
$$y_d(33) = y(33) - y_b(33) = -2{,}7 - 0{,}338\mu < 0$$

Como $y_d(33)$ é negativo (independentem. de μ) a estimativa do decisor é -1:

$$\hat{u}(33) = -1 .$$

d) $$e(33) = \hat{u}(33) - y_d(33) = -1 + 2{,}7 + 0{,}338\mu = 1{,}734$$
(se $\mu = 0{,}1$)

RLS, um coeficiente, $\alpha = 0{,}95$



$d(n) = a(n) - a(n-2) = 2, \pm 4$

$n=0$ e 1 $\quad$ $n>1$

a) $$\hat{R}(n) = \alpha \hat{R}(n-1) + a^2(n) \Rightarrow \hat{R}(n) = \alpha^n \hat{R}(0) + \sum_{i=1}^{n} \alpha^{n-i} a^2(i)$$

Como $a^2(i) = 4 \Rightarrow \hat{R}(n) = \alpha^n \hat{R}(0) + 4 \sum_{i=1}^{n} \alpha^{n-i}$. Mas $\sum_{i=1}^{n} \alpha^{n-i} = \sum_{k=0}^{n-1} \alpha^k = \frac{\alpha^n - 1}{\alpha - 1}$.

Substituindo valores: $\hat{R}(100) = 0{,}95^{100} \times 0{,}02 + 4 \times \frac{0{,}95^{100} - 1}{0{,}95 - 1} = 79{,}53$

(Se $tr[\underline{P}(0)] = 50 \Rightarrow P(0) = 50$ pois só temos escalares $(N=1)$, logo $\hat{R}(0) = \frac{1}{P(0)} = 0{,}02$.)

(6)

b) $e'(n) = d(n) - \underline{c}^T(n-1)\,\underline{a}(n)$

$\underline{c}(n) = \underline{c}(n-1) + \underline{k}(n)\,e'(n)$

$\underline{k}(n) = P(n)\underline{a}(n) = \dfrac{a(n)}{R(n)}$ (neste caso)

$n=1$ $R(1) = \alpha R(0) + a^2(1) = 0{,}95 \times 0{,}02 + 4 = 4{,}019$

$k(1) = \dfrac{a(1)}{R(1)} = \dfrac{2}{4{,}019} = 0{,}498$

$e'(1) = d(1) - \underbrace{c^*(0)}_{0}\,a(1) = d(1) = a(1) - a(-1) = 2 - 0 = 2$

[redacted] $c(1) = c(0) + k(1)\,e'(1) =$

$= 0 + 0{,}498 \times 2 = 0{,}995$

c)
$n=2$ $d(2) = a(2) - a(0) = -2 - 2 = -4$

$e'(2) = d(2) - c^*(1)\,a(2) = -4 - 0{,}995 \times (-2) =$

$= -2{,}009$.