Matching Lines in Image Sequences using Geometric Constraints

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Abstract – An approach for tracking lines along temporal image sequences is presented. Three independent Kalman filters are used in the process, taking either Mahalanobis normalised distances or geometric constraints as measures of the matching degree.

I - INTRODUCTION

In computer vision, the need to track lines in image sequences often occurs. For example, this need arises when the goal is to extract three-dimensional information from lines in the scene [1], or when the tracking of moving objects, modelled as wire frame structures, is desired.

In our approach, it is considered that:

- the entities to track are line segments, each one defined by the position of its midpoint, its direction, and its length.
- in the tracking process, three independent Kalman filters [2] are used: one for the midpoint position, one for direction, and one for the line length; all filters use a kinematics model of locally constant acceleration and a matching criterion based on Mahalanobis normalised distances or on geometric constraints. The use of three independent filters is justified by the fact that the parameters of the chosen line model are uncorrelated.
- a 2D model for the entities is used; when a new entity is visible, it is inserted in the model with a confidence factor of value 3; if this entity is visible again in the next image, its confidence factor is incremented, up to a maximum value of 5; on the other hand, if an entity in the model is not visible (that is, the matching fails) in an image, its confidence factor is decremented; if the confidence factor of an entity reaches the value zero, it is removed from the model. In this way, the elements of the 2D model are continually updated.

The following section addresses the parameterization of the entities. Then, the approach used for matching the entities in image sequences is described. After some experimental results, the main conclusions are drawn.

II - PARAMETERIZATION OF THE ENTITIES

The line entities require a convenient parameterization. One possible solution could be the use of the line endpoints; however, a major drawback of this approach is the fact that the lines may be visible differently in successive images [3]. Another solution might use for parameters the orientation θ, the length l, the distance c of the line to the frame origin, and the distance d between the line midpoint and the intersection of the line with its perpendicular through the origin [Fig. 1].

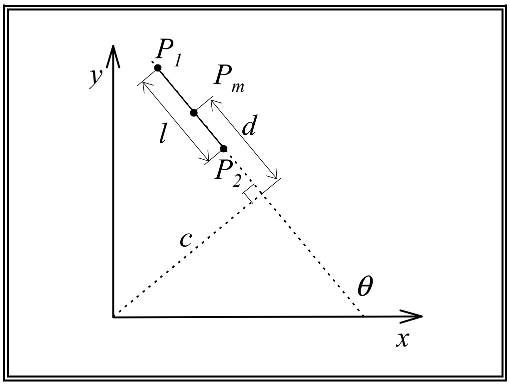

Fig. 1 - Parameterization θ, l, c and d, for line segments.

This approach presents some problems too: the parameters are correlated requiring the use of a single Kalman filter with proper dimension, which is computationally costly; the parameters c and d are strongly dependant on line position, making the matching process more difficult [3].

The parameter choice adopted in our work avoids the aforementioned problems; it consists on the use of the position of the line midpoint, its orientation, and its length. These parameters being uncorrelated, it is possible to use three independent Kalman filters: one for the coordinates, x_m and y_m, of the line midpoint;

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another for the orientation θ; and a last one for the length l. In this way, the computational efficiency is improved [3, 4].

III - MATCHING LINES IN IMAGE SEQUENCES

The kinematics model associated to each one of the three Kalman filters is a model of locally constant acceleration, considered as a first order Gauss-Markov process and equal to a fraction α of the previous acceleration value. The estimated acceleration tends to zero; if one entity disappears (that is, if it is not matched in subsequent images, causing high uncertainty) the filter will gradually reduce the last known acceleration by powers of α. This kinematics model is flexible and adaptable to other types of application.

As a measure of match, normalised Mahalanobis distances are used [2] or, when this method fails, geometric constraints are considered. Fig. 2 shows the approach used.

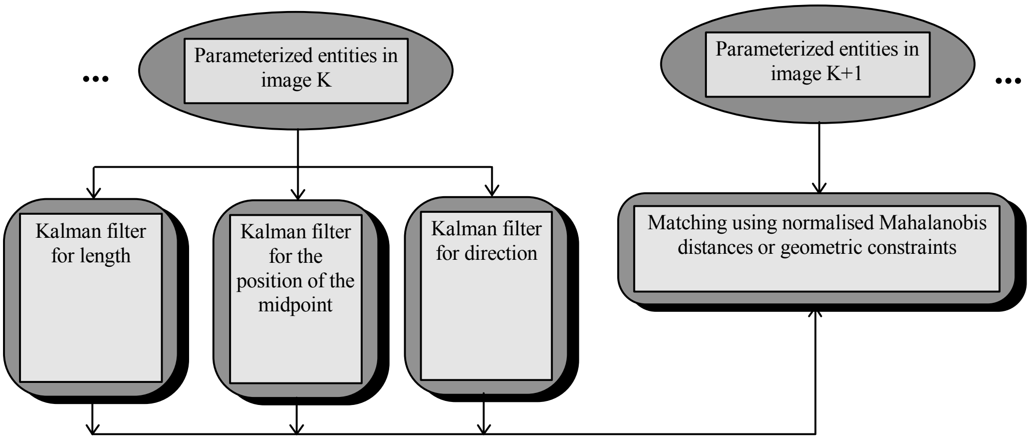


Fig. 2 - Matching of the entities in the adopted approach.

The Kalman filters, the normalised Mahalanobis distance, the geometric constraints, and the measurement and matching phase are described in the following subsections.

A - Kalman filter

The Kalman filter is used to estimate the values of the characteristics of the entities along time, that is, along the image sequence. The Kalman filter is a statistical approach to estimate a vector of time varying characteristics $\hat{x}_t$ from a set of noisy measures $\hat{z}_t$. It is a recursive scheme developed to describe the dynamic system model, the error statistics between model and reality, and the uncertainty associated to the measurement. The following phases may be considered:

- *Prediction:*

  The estimate of the characteristics vector $\hat{x}$ at time t is:

  $$\hat{x}_t^- = \Phi \hat{x}_{t-1}^+, \qquad \text{(Eq. 1)}$$

  where:
  - Φ is the kinematics matrix, a constant matrix involving the characteristics considered and their derivatives in subsequent time samples;
  - the superscripts $\pm$ designate the estimate after and before measurement, respectively.

  At time $t = 0$ the unknown characteristics are taken as null, and the other ones have the value determined by measurement.

  The estimate for the uncertainty associated to the vector of characteristics $\hat{x}$ at time t is:

  $$P_t^- = \Phi P_{t-1}^+ \Phi^T + Q, \qquad \text{(Eq. 2)}$$

where Q is a diagonal matrix, the elements of which are determined according to the application at hand, conveying the introduction in the model of the noise variance. In our application, the noise variance has been modelled as zero mean Gaussian, to avoid too much convergence of the Kalman filters.

At time $t = 0$ the uncertainty matrix is defined as diagonal with arbitrary values;

the values are, in fact, defined by experience.

- *Measurement and matching:*

  Having the estimates of the characteristics vector and of their uncertainties, their measurement in a given area is performed. Note that the derivatives need not be measured, as the Kalman filter allows their recursive estimation; furthermore, these estimates being given by integration, they are more immune to noise than the instantaneous derivatives computed by differences [5].

  The next step is to select, among the candidate entities, the one that has characteristics closer to the estimates. After this match, the vector of measured characteristics $\hat{z}_t$ assumes the values of the characteristics of the matched element, and the measured variances are assigned to the uncertainty matrix R_t.

- *Updating:*

  After measurement and matching, the gain K_t of the Kalman filter must be computed; this gain conveys the relative weight to assign to the measurement and to the estimate, and it is computed based on their relative uncertainties. The equation for the Kalman filter gain is:

$$K_t = P_t^- H^T \left[H P_t^- H^T + R_t\right]^{-1}, \qquad \text{(Eq. 3)}$$

where:

- H is the matrix that transforms the coordinate system of the estimated characteristics vector $\hat{x}_t$ into the coordinate system of the measured characteristics vector $\hat{z}_t$;
- R_t is the measured variance matrix.

Having the gain of the Kalman filter, the updating of the estimate is performed using the measurement. The updating is made according to:

$$\hat{x}_t^+ = \left[I - K_t H\right]\hat{x}_t^- + K_t \hat{z}_t, \qquad \text{(Eq. 4)}$$

where I is the identity matrix.

The correction of the uncertainty matrix is:

$$P_t^+ = \left[I - K_t H\right] P_t^- \left[I - K_t H\right]^T + K_t R_t K_t^T. \qquad \text{(Eq. 5)}$$

Note that, from Eqs. 4 and 5, that the updating assigns greater weight to the measurement relative to the estimate for larger Kalman filter gain; if the measurement uncertainty is smaller than the estimation uncertainty, the Kalman filter assigns greater weight to the measured values.

Fig. 3 shows the flow diagram of the Kalman filter.

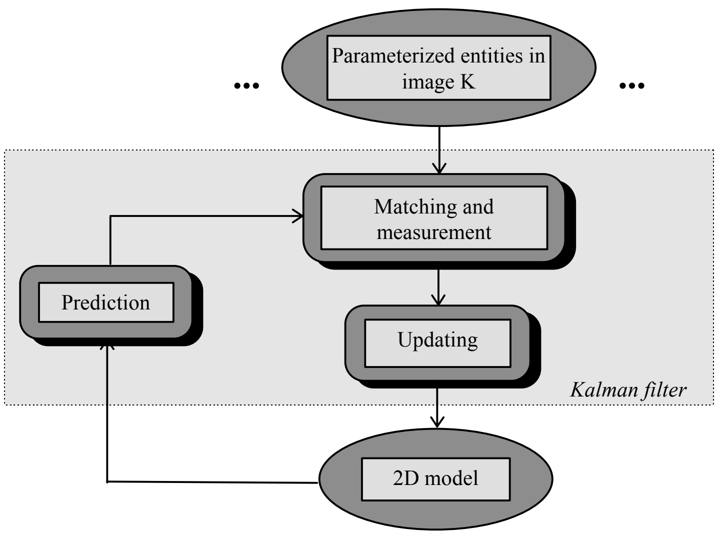


Fig. 3 - Flow diagram of the Kalman filter.

Note from Fig. 3 that when a new entity is initiated it must be updated; this is necessary to maintain the order measurement and matching - updating - prediction. Moreover, by doing this, the errors related to the initialisation assignments are reduced.

B - Normalised Mahalanobis distance

Inside the matching area given by the Kalman filter for midpoint position, it is necessary to define a measure of the match of each possible candidate to the entity characteristics estimated by the three Kalman filters. In our approach, normalised Mahalanobis distances are first used as match measures. The normalised Mahalanobis distance is defined as the difference of characteristics normalised by their variances, as:

$$d_{\chi^2_N} = \frac{(X_m - X_e)^T (V_m + V_e)^{-1} (X_m - X_e)}{2} \quad \text{(Eq. 6)}$$

where:

- X_m is the vector of characteristics and V_m is the matrix of variances of the matching candidate;
- X_e is the vector of characteristics and V_e is the matrix of variances, estimated by the corresponding filter.

In the case of a single scalar characteristic, the previous equation simplifies to:

$$d_{\chi^2_1} = \frac{(x_m - x_e)^2}{2(v_m + v_e)} \quad \text{(Eq. 7)}$$

where:

- x_m is the candidate characteristic and v_m its variance;
- x_e is the characteristic and v_e its variance, as estimated by the filter.

This distance has a χ^2 distribution, with a number of degrees of freedom equal to the number of characteristics considered. Thus, for example, when a 95% probability of successful match is desired and there is only one degree of freedom, the threshold for d_{χ^2} should be 3.841458.

C - Geometric constraints

When there is no successful match with the normalised Mahalanobis distances, a match is searched for again by using geometric constraints. These are:

- The difference between the length l_m of the candidate segment and the length l_e estimated by the respective Kalman filter, which must be less or equal to a given value MAX_{ladm}, that is:

$$d_l = \left| l_m - l_e \right| \leq MAX_{ladm}. \quad \text{(Eq. 8)}$$

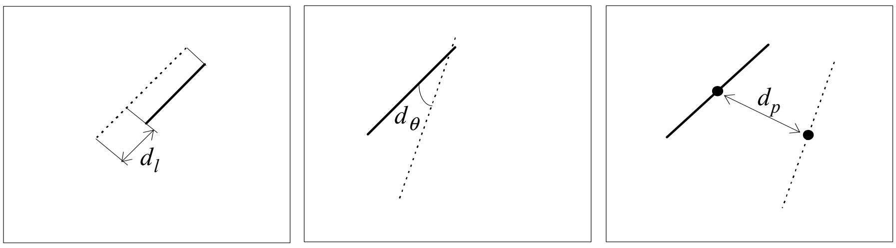
- The difference d_θ between the direction θ_m of the candidate segment and the direction θ_e estimated by the respective Kalman filter, which must be less or equal to a given value $MAX_{\theta adm}$, that is:

$$d_\theta = \left| \theta_m - \theta_e \right| \leq MAX_{\theta adm}. \quad \text{(Eq. 9)}$$

- The difference d_p between the position (X_m, Y_m) of the candidate midpoint and the position (X_e, Y_e) of the estimated midpoint given by the respective Kalman filter, which must be less or equal to a given value MAX_{padm}, that is:

$$d_p = \sqrt{(X_m - X_e)^2 + (Y_m - Y_e)^2} \leq MAX_{padm} \quad \text{(Eq. 10)}$$

Fig. 4 displays the geometric constraints used.



a) b) c)

Fig. 4 - Geometric constraints used: a) difference of lengths, b) difference of directions, and c) difference between midpoint positions. Solid lines represent the candidate segment characteristic and dotted lines the one predicted by the Kalman filter.

D - Measurement and matching phase

The estimation of the uncertainty matrix associated with the characteristics vector estimated by the Kalman filter for position determines an elliptical area, inside which the estimated entity must be. Defining matrix M by the equation:

$$M = HP_t^- \qquad \text{(Eq. 11)}$$

where, for the Kalman filter for midpoint position:

- H is the matrix that transforms the coordinate system of the estimated vector $\hat{x}_t$ into the coordinate system of the measured vector $\hat{z}_t$;
- P_t^- is the estimated uncertainty matrix;

it is possible to determine the elliptical matching area by considering that:

- the eigenvalues of matrix M define the major and minor lengths of the matching ellipse;
- the eigenvectors of matrix M define the major and minor axis of the ellipse.

This matching ellipse is, of course, centred on the midpoint position estimated. After the ellipse definition (see Fig. 5), it is necessary to perform the matching with the entity that has closer characteristics to the estimation.

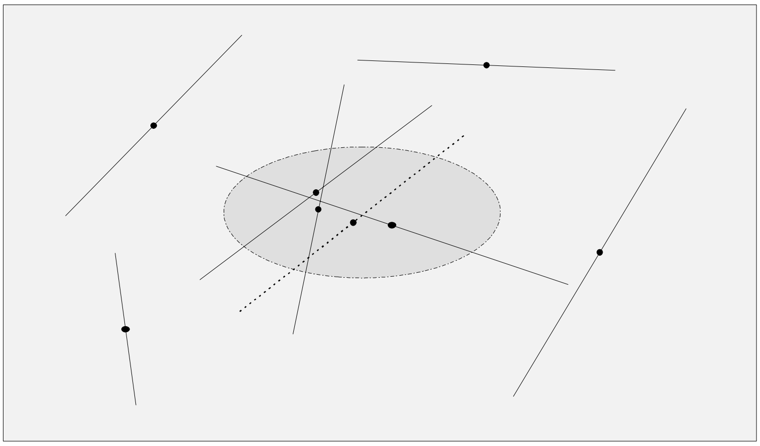


Fig. 5 - Ellipse resulting from the Kalman filter for midpoint position. Solid lines represent entities actually in the image, while the dotted line represents the 'entity' estimated by the three Kalman filters.

In a first step, matching is done according to the following algorithm:

Begin

{

Take as candidate entities all those which have a midpoint inside the matching ellipse and that were not matched before.

If there is no such candidate **then end**.

Do for all matching candidates:

{

Compute the Mahalanobis distance $d_{\chi_1^2}(\theta_i, \theta_e)$ between direction θ_i of the candidate and the Kalman filter predicted direction θ_e.

If the computed distance is larger than a specified threshold **then continue**.

Compute the Mahalanobis distance $d_{\chi_1^2}(l_i, l_e)$ between the length l_i of the matching candidate and the length l_e predicted by the corresponding Kalman filter.

If the computed distance is larger than a specified threshold **then continue**.

Compute the Mahalanobis distance $d_{\chi_2^2}((x,y)_i, (x,y)_e)$ between the midpoint position $(x,y)_i$ of the matching candidate and the one predicted by the corresponding Kalman filter $(x,y)_e$.

If the computed distance is larger than a specified threshold **then continue**.

Compute the product of the three distances previously computed.

If the computed product is smaller than the current minimum **then** take the current candidate as the best match[1].

}

}

If matching is successful then the measured state is the following:

- ***For the direction Kalman filter:***
The direction element of the measured characteristics vector assumes the value of

[1] For better matching results special care must be taken in the choice of the distance thresholds, in order that the absorbing property of the multiplication may not produce erroneous matches.

the direction of the matched entity θ_i; the element corresponding to the direction variance in the uncertainty matrix associated with measurements, assumes the value of the variance computed inside the matching ellipse.

- ***For the length Kalman filter:*** The length element of the measured characteristics vector assumes the value of the direction of the matched entity l_i; the element corresponding to the length variance in the uncertainty matrix associated with measurements, assumes the value of the variance computed inside the matching ellipse.
- ***For the midpoint position Kalman filter:*** The elements of the measured characteristics vector corresponding to the midpoint position coordinates x and y assume the values of the matched entity x_i and y_i; the elements in the uncertainty matrix associated with measurements, corresponding to the variances s_{xx}, s_{xy}, s_{yy} and s_{yx}, assume the respective values computed inside the matching ellipse.

When matching is not successful up to this point, an attempt is made to perform matching based on geometric constraints, as follows:

Begin

{

Do for all unmatched entities:

{

Compute the difference d_θ between direction θ_i of the candidate entity and the one predicted by the filter θ_e.

If d_θ is larger than the threshold **then continue**.

Compute the difference d_l between length l_i of the candidate entity and the one predicted by the filter l_e.

Se d_l is larger than the threshold **then continue**.

Compute the difference d_p between the midpoint position p_i of the candidate entity and the one predicted by the filter p_e.

Se d_p is larger than the threshold **then continue**.

Compute the product $d_\theta d_l d_p$.

If the product is smaller than the current minimum **then** take the current candidate as the best match[2].

}

}

When a match is achieved, the measured state is computed as before, but centring the ellipse at the midpoint of the matched entity.

If matching is not achieved, that being caused by the permanent or temporary absence of the entity, the measured state is assumed as:

- ***For the direction Kalman filter:*** The direction element of the measured characteristics vector assumes the value of the direction estimated by the respective Kalman filter θ_e; the element corresponding to the direction variance in the uncertainty matrix associated with measurements, assumes the value assigned for initialisation.
- ***For the length Kalman filter:*** The length element of the measured characteristics vector assumes the value of the direction estimated by the respective Kalman filter l_e; the element corresponding to the length variance in the uncertainty matrix associated with measurements, assumes the value assigned for initialisation.
- ***For the midpoint position Kalman filter:*** The elements of the measured characteristics vector corresponding to the midpoint position coordinates x and y assume the values estimated by the respective Kalman filter x_e and y_e; the elements s_{xx} and s_{yy} of the uncertainty matrix associated with measurements, assume the values assigned for initialisation.

In this way, when matching fails the uncertainty of the measurement is increased, so that the Kalman filters give a larger weight to the predictions by reducing their gain. In the next prediction, the area of the matching ellipse is larger.

Note that for small differences among successive images the matching process is easier and so is the tracking of lines. This means that the movement of the lines should not change too

[2] Recall the previous footnote.

much from one image to the next, especially when the Kalman filters have already converged. The use of geometric constraints, however, reduces this requirement.

IV - EXPERIMENTAL RESULTS

For the testing of the adopted approach, a sequence of 12 images having 10 lines was synthetically generated (other experimental results, including real cases, are presented in [1]). Some of the images in the sequence are shown in Figs. 6-8. Note that not all lines are present in each image; this was done to simulate the possibility that some lines may be invisible in an image and then visible again in the next image and vice versa.

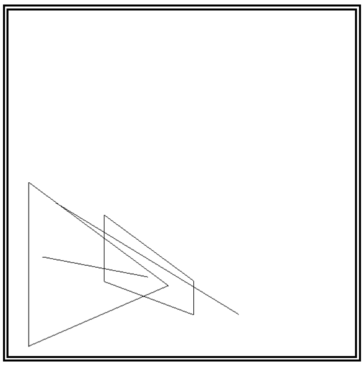


Fig. 6 - Image 1 of the test sequence.

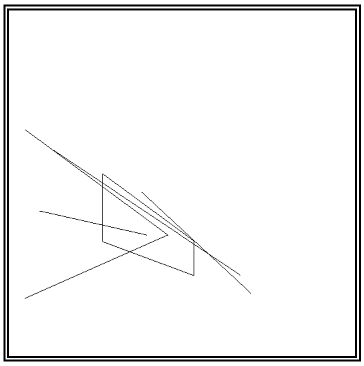


Fig. 7 - Image 6 of the test sequence.

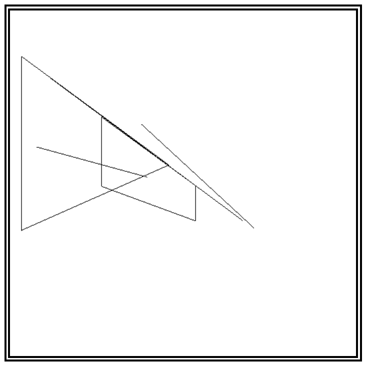


Fig. 8 - Image 12 of the test sequence.

Fig. 9 shows the image of the superposition of the first and last image in the sequence, where the total displacement of the lines is clearly discernible.

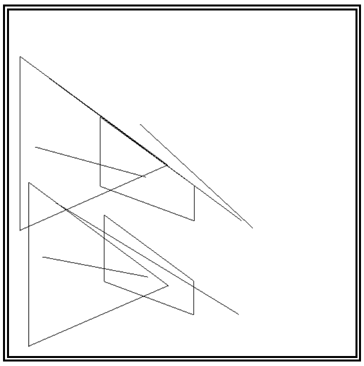


Fig. 9 - Superposition of the first and last images in the test sequence.

Table I, at the end of the paper, presents some results achieved for the test sequence, showing how the tracking process has occurred for two of the lines, using the current implementation (described in detail in [6]).

V - CONCLUSIONS

An approach for tracking lines in image sequences has been presented. The line parameters are the midpoint position, the length and the direction. Three independent Kalman filters are used for tracking, one for each parameter. The matching measure between the characteristics of a candidate line and the ones predicted by the filters is the normalised Mahalanobis distance or, if matching fails, geometric constraints are tested. The kinematics model for all filters is one of locally constant acceleration.

The approach used offers good results, as evident in the experimental results shown. The use of geometric constraints facilitates the tracking, as expected. The measurement and updating techniques adopted proved correct, enabling the tracking of lines even in the cases where they vanish and then reappear. It was also observed that when the Kalman filters start diverging or converging 'in excess', the use of the geometric constraints allows their faster self-adjustment.

In the experiments, it has been found that the Kalman filters could converge excessively, making the matching ellipses too small, thus preventing successful matches. The solution for this problem was the introduction of noise in the model, although the use of geometric constraints reduces the occurrence of these situations, as previously stated. The initialisation of the model is also made more flexible by the geometric constraints.

The use of a confidence factor for the entities in the model has proved useful, namely by keeping the model updated and avoiding it to grow unnecessarily.

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Table I - Results of the tracking process for the test sequence.

Image 1						**Image 2**					
Ref.	***St. P.***		***End P.***		***Mtc.***	***Ref.***	***St. P.***		***End P.***		***Mtc.***
1	26	256	235	411	I	1	24	240	235	396	KM
2	235	411	26	502	I	2	235	396	25	488	KM
Image 3						**Image 4**					
Ref.	***St. P.***		***End P.***		***Mtc.***	***Ref.***	***St. P.***		***End P.***		***Mtc.***
1	23	225	235	381	KM	1	Not visible				
2	235	381	24	474	KM	2	235	366	23	459	KM
Image 5						**Image 6**					
Ref.	***St. P.***		***End P.***		***Mtc.***	***Ref.***	***St. P.***		***End P.***		***Mtc.***
1	21	193	235	351	KM	1	20	177	234	335	KM
2	235	351	21	445	KM	2	234	335	20	430	KM
Image 7						**Image 8**					
Ref.	***St. P.***		***End P.***		***Mtc.***	***Ref.***	***St. P.***		***End P.***		***Mtc.***
1	19	161	234	320	KM	1	18	128	234	288	KGC
2	234	320	19	415	KM	2	234	288	18	384	KGC
Image 9						**Image 10**					
Ref.	***St. P.***		***End P.***		***Mtc.***	***Ref.***	***St. P.***		***End P.***		***Mtc.***
1	16	111	234	272	KGC	1	15	95	234	256	KM
2	234	272	16	369	KGC	2	234	256	15	353	KM
Image 11						**Image 12**					
Ref.	***St. P.***		***End P.***		***Mtc.***	***Ref.***	***St. P.***		***End P.***		***Mtc.***
1	14	78	234	240	KM	1	13	67	234	230	KM
2	234	240	14	337	KM	2	234	230	13	328	KM

Legend:

Ref. - Reference of the line, **I -** Initialisation of the line, **St. P.** - Image coordinates of the starting point, **End P.** - Image coordinates of the end point, **Mtc.** - Type of matching: **KM** - Kalman filter and Mahalanobis distances, **KGC** - Kalman filter and geometric constraints.